



RESEARCH DEPARTMENT

Photographic film grain: a study with the aid of an optical correlator

REPORT No. T-101

1963/5

**THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION**

RESEARCH DEPARTMENT

PHOTOGRAPHIC FILM GRAIN:
A STUDY WITH THE AID OF AN OPTICAL CORRELATOR

Report No. T-101

(1963/5)

K. Hacking, B.Sc.

D. Maurice

(D. Maurice)

This Report is the property of the
British Broadcasting Corporation and
may not be reproduced in any form
without the written permission of the
Corporation.

PHOTOGRAPHIC FILM GRAIN:

A STUDY WITH THE AID OF AN OPTICAL CORRELATOR

Section	Title	Page
	SUMMARY	1
1.	INTRODUCTION	1
2.	FUNDAMENTAL IDEAS	3
3.	METHODS OF MEASURING GRANULARITY	6
4.	THEORY OF THE AUTOCORRELATION APPROACH	8
5.	APERTURE WEIGHTING AND GRANULARITY TRANSFER	11
	5.1. Effect of Aperture Function	12
	5.2. Positive Printing	14
6.	EXPERIMENTAL TECHNIQUE	17
	6.1. The Photomicroscope	18
	6.2. Possible Errors in the Photomicrography	20
	6.2.1. Limitations of the Microscope Objective	20
	6.2.2. Photographic Processing	20
7.	THE OPTICAL CORRELATOR	21
	7.1. Description of the Instrument	21
	7.1.1. Optical Design	21
	7.1.2. Mechanical Design	22
	7.1.3. Electrical Arrangement	23
	7.2. Measurement Procedure	23
	7.3. Performance Tests	24
	7.4. Interpretation of the Measured Function	25

Section	Title	Page
8.	RESULTS	27
8.1.	Comparison of the Autocorrelation Functions and Wiener Spectra of the Six Emulsions	27
8.2.	Variation of the Autocorrelation Function, Mean-Signal-to-r.m.s. Noise Ratio and Equivalent Grain Diameter with Mean Transmission	28
8.3.	Effect of Development on Granularity	32
8.4.	Callier Quotient Measurements	36
9.	DISCUSSION OF RESULTS	37
9.1.	Comparison of Results for the Six Emulsions	37
9.2.	Variation of Granularity with Mean Transmission	39
9.3.	Negative Correlation	40
9.4.	Effect of Development on Granularity	41
9.5.	Callier Quotient	41
10.	CONCLUSIONS	42
11.	REFERENCES	42

January 1963

(1963/5)

PHOTOGRAPHIC FILM GRAIN:

A STUDY WITH THE AID OF AN OPTICAL CORRELATOR

SUMMARY

The autocorrelation and power-spectrum approach to the problem of describing near-random functions of time in electrical communication systems is applied to the objective description of the underlying granular structure of photographic images. The analogy between random electrical noise and emulsion granularity seems even more direct when considering television applications of film, since at one stage (after scanning) the transferred granularity exists as a single function of time and, indeed, may be indistinguishable in character from electrically generated noise. However, the two-dimensional nature of photographic grain must be taken into account when dealing with the general analysis of scanning with finite apertures and of granularity transfer, both of which are discussed in this report. Simple representative models of the grain structure are used as a guide to the basic properties of film grain and serve as useful standards for comparison with experimental results.

The grain autocorrelation functions of several photographic emulsions, typical of those used in television film processes, have been measured using an optical correlator. The construction of the correlator, its performance, and the techniques employed are described in some detail.

Variations in the properties of emulsion grain with such parameters as optical density and development time have been briefly investigated. The numerical assessment of the signal-to-r.m.s. noise ratio due to granularity, and the estimation of its subjective visibility, when using these emulsions, or combinations of these emulsions, in specific television applications, will be the subject of a further report.

1. INTRODUCTION

The microscopical examination of an exposed and developed photographic emulsion shows that the image consists of an agglomeration of silver particles or grains which are substantially opaque to visible radiation and are separated by relatively clear areas. The greater the ratio of opaque to clear area the greater is the "light-stopping" power: spatial fluctuations of the initial exposure are thus recorded as corresponding spatial variations of the population-density of the developed grains. Enlargement of the recorded image often partially reveals the underlying granular structure, and the subjective "graininess" of the picture is usually regarded as detrimental to picture quality. Objective measurements of the grain structure in emulsions are often referred to as "granularity" measurements, and this term will be used in this report.

Scientific workers, especially those connected with the film industry, have been interested in studying and measuring the properties of the developed silver image for at least the last four decades and the volume of literature on various aspects of the subject is overwhelming. Perhaps one could say that the main aim of a great deal of previous work has been to obtain a one-parameter measure or description of emulsion granularity which correlates sufficiently well with the subjective estimation of graininess in the final print or projected picture. Several important researches (prior to 1945) into the objective evaluation of emulsion granularity were reviewed in detail in a series of papers by Jones and Higgins.¹ The agreement between the results obtained by the various methods used (mainly based on scanning techniques) was not always satisfactory. It seemed clear, to quote from the introduction to the second paper by Jones and Higgins, that "the objective methods studied measured some, but not all, of the factors controlling graininess".

Quantitative granularity measurements are (as, indeed, are ordinary measurements of density and development "gamma") extremely dependent upon the particular optical arrangement of the instrument used for the measurement. Such instrumental dependencies and other practical difficulties, such as the preparation of "clean" samples, tend to obscure the underlying fundamental character of granularity and the possible agreement between the results obtained by various techniques. Again, some workers have chosen to measure granularity in terms of deviations of transmission² others in terms of deviations of density³ ($= -\log \text{transmission}$) and it is not always straightforward to convert from one to the other with sufficient accuracy to make realistic comparisons of the results.

A different approach to the problem of specifying the granularity of an emulsion has been gaining ground during the last decade and makes use of the statistical correlation and Fourier analysis techniques which have already proved to be valuable tools in electrical communication theory. To apply these techniques a knowledge of either the autocorrelation function of the emulsion grain structure or its Fourier spectrum is required. Objective measures of noise such as signal-to-noise ratio and noise power spectrum, which have been used extensively in the assessment of random electrical noise, are directly applicable to emulsion granularity, the only difference being the two-dimensional nature of the grain structure.

P. Fellgett⁴ advocates the direct measurement of the autocorrelation function while R. Clark Jones⁵ regards the Wiener spectrum (analogous to noise power spectrum in electrical theory) as the most elegant form in which to specify granularity. The theoretical basis of these newer concepts has been well formulated in recent years, but published experimental results seem to be lacking.

The basic technique and type of optical autocorrelator used in the investigation reported here appear to have been first used by Selwyn and Coates⁶ in 1953 and their paper includes examples of grain autocorrelation functions. However, their method does not seem to have been pursued or further developed.

The precision optical correlator described in this report was designed for general research on picture correlation. One application is the study of the granularity of uniformly exposed and developed photographic films, and an experimental technique was developed in order to obtain the Wiener spectra of several commercially available emulsions* often used in telecine and telerecording processes.

*Only black and white emulsions have been investigated.

2. FUNDAMENTAL IDEAS

A simple physical model of the developed emulsion can be devised from which the general properties of granularity can be deduced. For instance, one elementary form is a checkerboard (two-dimensional) arrangement of uniform-size "black" areas randomly positioned. Albersheim⁷ made a statistical theoretical analysis using such a model and extended the analysis to a three-dimensional array. Webb⁸ used a checkerboard of square (and circular) dots randomly arranged for studying the statistical properties of grain. Recently, Marchant and Dillon⁹ have simulated grain patterns by photographing electrical noise displayed on a cathode-ray tube. These studies provide a reasonable theoretical basis for some of the main properties observed with actual emulsions, but the initial assumptions may not be sufficiently realistic to account for other practical observations. For instance, in many actual emulsions the developed silver halide grains are certainly not uniform in size (i.e., either in volume or projected area) and there is, also, no *a priori* reason to assume that their spatial distribution is truly random. Again, individual grains may not be uniformly opaque to visible radiation: the electron micrography of single developed grains shows that they can be filamentary in structure,¹⁰ rather like a piece of sponge. An important factor, which influences practical measurements of density, etc., is the scattering of some of the incident light which takes place. The mechanism of the scattering is complicated, especially in thick emulsions which consist of many grain layers with considerable overlapping of grains. Further, since the average linear dimension of the individual grains is of the order of one to five wavelengths of light, depending upon the emulsion, it is clear that diffraction effects cannot be entirely disregarded.

In spite of the possible shortcomings of a simple physical model it is instructive to examine its basic properties. Suppose we consider a uniformly exposed and developed emulsion to be simply a layer consisting of a three-dimensional mosaic of opaque grains embedded in a transparent medium. A section through the hypothetical emulsion layer is illustrated in Fig. 1(a). Let us make two assumptions only at this stage:

- (1) that the grains are uniformly opaque;
- (2) that light traversing the layer is not diffracted.

Then the structure will resolve (in any direction) into a two-dimensional array of opaque elements each with infinitely sharp edges. Consider the layer to be uniformly illuminated in a direction normal to the layer and set up rectangular co-ordinate axes (x, y) in the surface of the layer as shown in Fig. 1(b). Let the function $t(x, y)$ represent the transmission coefficient, normal to the layer, at any point (x, y) on the

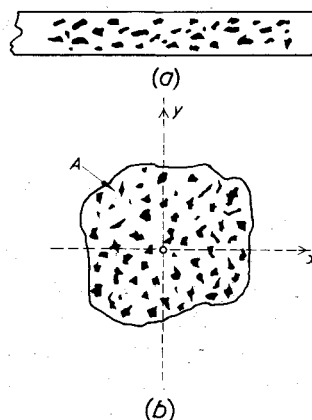


Fig. 1 - Simple model of the grain structure of a developed emulsion

- (a) Normal section through the layer
- (b) Projection normal to the surface of a region "A" showing rectangular co-ordinate axes x and y

surface, then the variance, σ^2 , of $t(x,y)$ over a prescribed region A which includes the origin is:

$$\sigma^2 = A^{-1} \iint_{(A)} t^2(x,y) dx dy - (\bar{t})^2 \quad (1)$$

where $\bar{t}$ is the mean transmission coefficient over the region A . But with the above assumptions $t(x,y)$ can only take on the eigenvalues 0 and 1 so that we have:

$$\sigma^2 = \bar{t} - (\bar{t})^2 = \bar{t} (1 - \bar{t}) \quad (2)$$

and the root mean square deviation σ , is:

$$\sigma = \{\bar{t} (1 - \bar{t})\}^{\frac{1}{2}} \quad (3)$$

Equation (3) thus shows that the r.m.s. value of the deviations in transmission from the mean is only a function of the mean transmission. σ is clearly a maximum when $\bar{t} = 0.5$, that is when the projected area of the opaque elements in A is equal to the clear area. Note that the result given by equation (3) is independent of the size or spatial distribution of the opaque elements. It will be seen later that a more convenient parameter is the ratio of the mean transmission to its r.m.s. deviation. From equation (3) we obtain:

$$\frac{\bar{t}}{\sigma} = \left(\frac{\bar{t}}{1 - \bar{t}} \right)^{\frac{1}{2}} \quad (4)$$

This function is zero for $\bar{t} = 0$ and infinite for $\bar{t} = 1$. Equations (3) and (4) are plotted in Fig. 2. The above result means, in physical terms, that if an area (supposed large compared with the average size of the opaque grains) of the grain layer is uniformly illuminated and then scanned with a "pinhole" aperture the ratio of mean transmitted flux to r.m.s. deviation (mean-signal-to-r.m.s. noise ratio) will approach the value given by equation (4) as the scanning aperture becomes vanishingly small. A point of interest is that no other model of the same mean transmission can be devised which will have a mean-signal-to-r.m.s. noise ratio less than that given by the right-hand side of equation (4), so that this function represents a fundamental lower limit and is independent of the size and distribution of the grains. It is only when we scan the area with an aperture of finite size that the size and spatial distribution of the grains must be taken into account. This will appear more obvious later when we consider both the Fourier spectrum of the signal fluctuations and of the scanning aperture.

Suppose, now, we adjust the physical model slightly so that the function $t(x,y)$ can take on the eigenvalues 1 and k instead of 1 and 0 as previously. (This may be achieved hypothetically by adding a uniform "flare" to the system which reduces the grain contrast from ∞ to $1/k$.) In this case it may be shown that the minimum ratio of mean transmission to its r.m.s. deviation is:

$$\left(\frac{\bar{t}}{\sigma} \right)_k = \left(\frac{\bar{t}}{1 - \bar{t}} \right)^{\frac{1}{2}} \cdot \left(\frac{\bar{t}}{\bar{t} - k} \right)^{\frac{1}{2}}, \quad 0 \gg k \gg \bar{t} \quad (5)$$

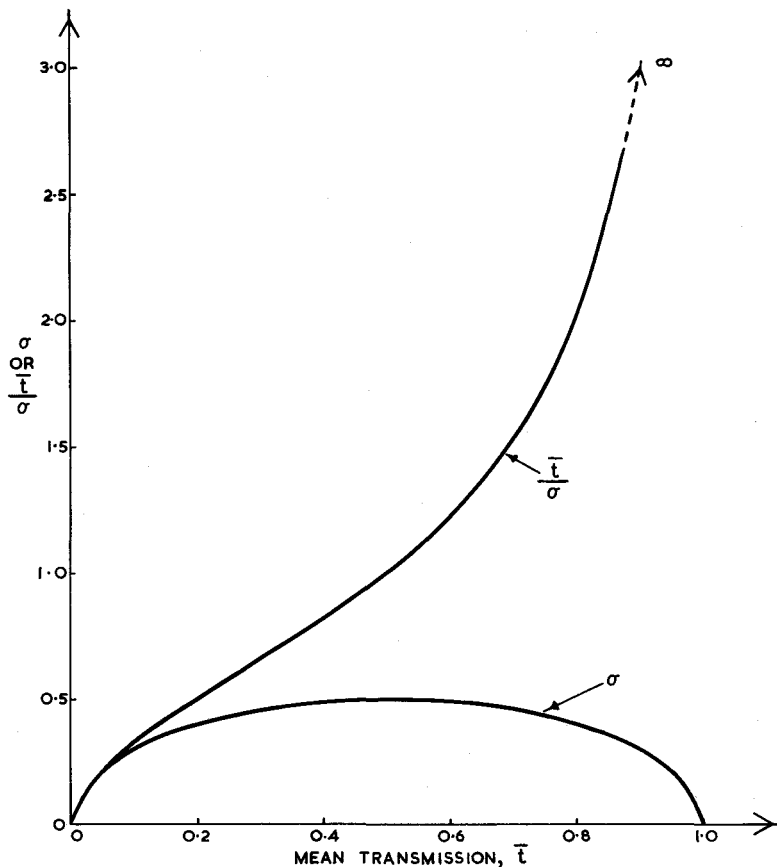


Fig. 2 - Fundamental limiting values of σ and $\bar{E}/\sigma$ as a function of mean transmission, E

which is seen to be the ratio for infinite grain contrast multiplied by the factor $\{\bar{E}/(\bar{E} - k)\}^{\frac{1}{2}}$. Equation (5) is plotted in Fig. 3 for $k=0$, 0.03 and 0.1 respectively, these particular values being arbitrarily chosen. The effect of reducing the grain contrast is to increase the limiting value of the mean-signal-to-r.m.s. noise ratio: the relative increase is small at high values of the mean transmission but becomes considerably greater at low mean transmissions. The ratio $(\bar{E}/\sigma)_k$ is always non-zero when k is non-zero and, in particular, for $\bar{E} = k$ the ratio is infinite.

So far, only the fundamental lower limit of the statistical measure of the transmission fluctuations has been discussed. Any optical device used to measure or "read off" the information recorded on a photographic emulsion performs, in effect, a scanning process with an aperture of finite size. For example, the visual perception of a photographic print can be explained as the effective scan of the picture by an aperture which is the projection of a retinal cone through the eye lens on to the plane of picture. Jones¹¹ has shown this concept to be useful in the visual assessment of print graininess. Clearly, the smaller the effective aperture is made the finer is the recorded detail which (if present) might be expected to be seen or resolved. At the same time, however, the smaller the effective scanning aperture the larger will be the fluctuations in intensity due to granularity. Eventually a limit is reached where the recorded detail information is swamped by the granularity. It is obvious, therefore, that the granularity of the emulsion is an important factor in assessing its capacity to record and convey information.

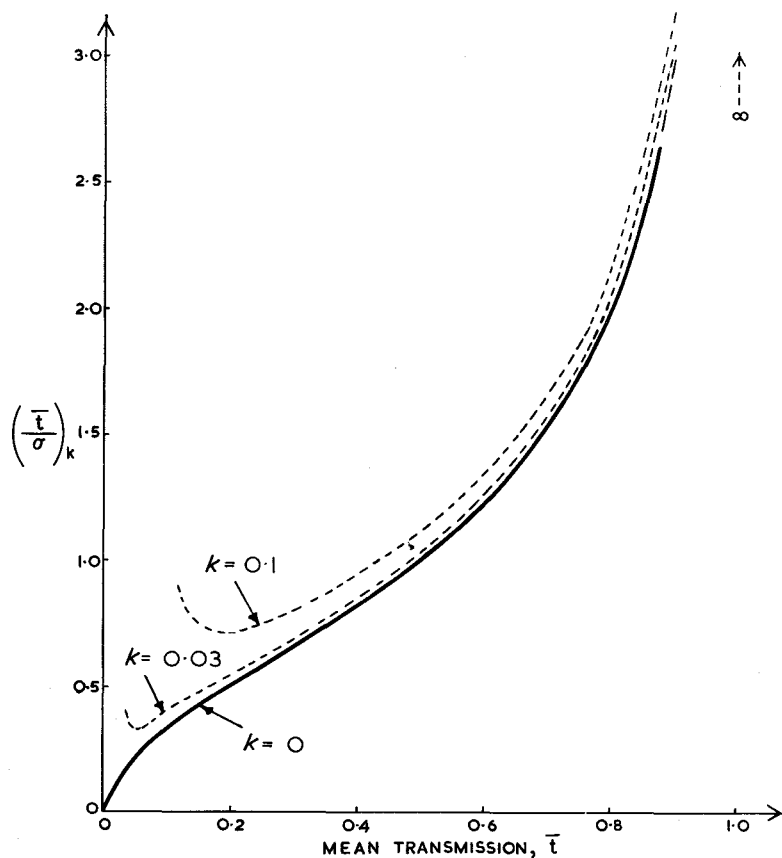


Fig. 3 - Effect on the limiting value of $(\bar{t}/\sigma)_k$ of uniformly reducing the grain contrast from ∞ to $1/k$

3. METHODS OF MEASURING GRANULARITY

There are several methods which have been used or proposed for measuring and describing emulsion granularity. A brief outline of some of these "measures" is given below, and it may be shown¹² that, in principle, they are inter-convertible since each contains the same basic information:

(i) The standard deviation in density or transmission when the uniformly exposed and developed film is scanned with a single aperture. Selwyn³ found that the fluctuations in density during such a scan followed a Gaussian distribution and showed that the standard deviation was inversely proportional to the square root of the aperture area. Thus the product of standard deviation and the square root of the scanning area will be constant for a given film and may be used as a one-parameter measure of its granularity. This measure, known as the Selwyn granularity constant, was however found to depend to some extent on the size of the scanning aperture, especially when the latter became very small, suggesting that the Gaussian distribution does not strictly hold when the average fluctuations in density are large. Recent work¹³ on carefully prepared specimens showed that the Selwyn relationship holds for a wide range of aperture size (approximately 10 to 300 microns in diameter).

However, a complete description of granularity by this method requires a knowledge of the Selwyn granularity constant as a function of aperture size. A similar result² is obtained if transmission, rather than density, fluctuations are measured providing the fluctuations are relatively small. As a method of comparing the granularity of different emulsions and for process control in emulsion manufacture the Selwyn measure is very convenient, and precision scanning microdensitometers have been constructed for this purpose.

(ii) The standard deviation of the density or transmission differences between two separated apertures scanning the film. (These differences are known as "syzygetic" differences.)

This is a very similar method to the above and the standard deviation is found to depend on the area of the apertures in much the same way.¹²

(iii) The autocorrelation function of the grain. In this method the average product (over the sample area) of the transmission coefficient at a point (x, y) in the surface with the transmission coefficient at a neighbouring point $(x + k, y + l)$ is measured as a function of k and l . The autocorrelation function so obtained contains the required granularity information in its most basic form, as will be seen later. Several instrumental techniques have been proposed for obtaining the basic autocorrelation function all of which present difficulties due to the small dimensions of the grains if high accuracy is desired. From the basic autocorrelation function the effect of scanning the film with prescribed aperture functions can be deduced.

(iv) The Wiener spectrum. This is the square of the modulus of the Fourier spectrum of the transmission function of the film. It is analogous to the noise power spectrum of an electrical signal. A well-known result in electrical communication theory is that the power spectrum and the autocorrelation function of the noise are reciprocal Fourier transforms. In general both the grain autocorrelation function and its transform, the Wiener spectrum, are two-dimensional functions. Fortunately it is found that ordinary emulsions have isotropic statistical properties so that the autocorrelation function can be described sufficiently by its generating curve, and the numerical two-dimensional transform is greatly simplified.

It is possible to measure directly the modulus of the frequency/response function and so obtain the Wiener spectrum without recourse to Fourier transformation of the autocorrelation function, but this, to the writer's knowledge, has not yet been done for emulsion grain. Having obtained the Wiener spectrum it is again straightforward to deduce the effects of either scanning with prescribed aperture functions or spatial-frequency bandwidth restrictions imposed by the low-pass filtering effect of subsequent transfer processes, (e.g., printing to obtain a positive record). In the writer's view the analysis of noise in systems involving photographic film is explained and understood more clearly in terms of the Wiener spectra of the emulsions and the overall frequency response of the systems.

(v) The average grain size. H. Frieser¹⁴ has shown that the sensitivity of an emulsion, for daylight exposures, is roughly proportional to the average grain volume. In general, therefore, the average projected area of the developed grains will be larger for the faster emulsions. If the spatial distribution of the developed grains is truly random in all uniformly exposed emulsions then it follows that the statistical

frequency distribution of the size of the grains will be the basic factor controlling their relative granularity. In those instances where the grain-size frequency distribution of an emulsion can be specified sufficiently by its mean value the latter provides a useful measure of granularity.

Eggert and Kuester¹⁵ found a relationship between the average grain diameter, d , and the Callier quotient, Q , (ratio of specular to diffuse density) of the form:

$$d \propto \log Q \quad (6)$$

This result is supported by the fact that Q was found to be independent of the spatial distribution of the grains. Callier quotient measurements thus provide an indirect method of estimating the average developed grain size. The method has the advantage that the Callier measurement can be made directly on the original developed emulsion.

In emulsion research the average grain size is usually determined by one of the standard methods of particle-size analysis¹⁶ which involve counting the number of grains in a given volume. These techniques can be used, also, to determine the size frequency distribution.

It will be shown later that the average projected grain size can also be estimated from the autocorrelation function.

4. THEORY OF THE AUTOCORRELATION APPROACH

As previously, let $t(x,y)$ represent the transmission function of a uniformly exposed and developed emulsion where the co-ordinate axes (x,y) lie in the plane surface of the emulsion. Let $\bar{t}_A$ be the mean transmission coefficient of a region of the surface of area A which includes the origin of co-ordinates. The grain autocorrelation function, ϕ , is defined as:

$$\phi(k,l) \equiv \lim_{A \rightarrow \infty} \left\{ A^{-1} \iint_{(A)} t(x-k, y-l) t(x,y) dx dy - \bar{t}_A^2 \right\} \quad (7)$$

where (k,l) are general displacement co-ordinates corresponding to the directions (x,y) respectively. The normalized autocorrelation function is given by $\phi(k,l)/\phi(0,0)$, the normalizing factor $\phi(0,0)$ being, in fact, the variance of the transmission function (c.f. equation (1), Section 2).

It will be seen that $\phi(k,l)$ is a two-dimensional even function having a maximum value at $\phi(0,0)$ and can be represented by a solid figure. According to the Wiener-Khinchine theorem the Fourier transform of the autocorrelation function is the square of the modulus of the generalized Fourier transform of the transmission function. Formally we can write,

$$\phi^*(k,l) = |G(f_x, f_y)|^2$$

$$\text{and} \quad |G(f_x, f_y)|^2 \equiv \lim_{A \rightarrow \infty} A^{-1} \left| \iint_{(A)} \{ t(x,y) - \bar{t}_A \} e^{-2\pi i (x f_x + y f_y)} dx dy \right|^2 \quad (8)$$

where * denotes the Fourier transform and $G(f_x, f_y)$ is the Fourier spectrum of the departures or fluctuations of the transmission function from its mean value, f_x and f_y being spatial frequency co-ordinates corresponding to the directions x and y respectively. The right-hand side of the second equation (8) has been called the Wiener spectrum $N(f_x, f_y)$, so that we have, since $\phi(k, l)$ is a real and even function:

$$N(f_x, f_y) \equiv |G(f_x, f_y)|^2 = \iint_{(\omega)} \phi(k, l) \cos 2\pi (kf_x + lf_y) dk dl \quad (9)$$

and, also, the inverse relationship:

$$\phi(k, l) = \iint_{(\omega)} N(f_x, f_y) \cos 2\pi (kf_x + lf_y) df_x df_y \quad (10)$$

As previously stated, the variance σ^2 , of the transmission function $t(x, y)$ is given by $\phi(0, 0)$ hence, using equation (10):

$$\sigma^2 = \phi(0, 0) = \iint_{(\omega)} N(f_x, f_y) df_x df_y \quad (11)$$

This important relation states that the variance of the transmission function is equal to the volume under the Wiener spectrum.

If the grain structure of the emulsion is isotropic, in terms of its statistical properties, the solid representation of both $\phi(k, l)$ and $N(f_x, f_y)$ will have rotational symmetry. These functions can then be described by the simpler functions, $\phi(s)$ and $N(f)$ respectively, which are the generating curves, where

$$f = (f_x^2 + f_y^2)^{\frac{1}{2}}$$

$$\text{and } s = (k^2 + l^2)^{\frac{1}{2}}$$

Consequently, introducing polar co-ordinates in equations (9) and (10) leads to the corresponding simplified equations for isotropic granularity:

$$\phi(s) = 2\pi \int_0^\infty J_0(2\pi fs) N(f) f df \quad (12)$$

$$\text{and } N(f) = 2\pi \int_0^\infty J_0(2\pi fs) \phi(s) s ds \quad (13)$$

where J_0 is the zero-order Bessel function of the first kind. Using tables of Bessel functions, $N(f)$ can now be computed from $\phi(s)$ (or *vice versa*) by straightforward numerical integration.

To illustrate the relationship between $\phi(k, l)$ and $N(f_x, f_y)$ a typical grain autocorrelation function and its two-dimensional Fourier transform are shown in Figs. 4(a) and 4(b) respectively. In this example

$$\sigma^2 = 0.1 = \phi(0, 0)$$

$$\text{and } \phi(0, 0) = \iint_{(\omega)} N(f_x, f_y) df_x df_y$$

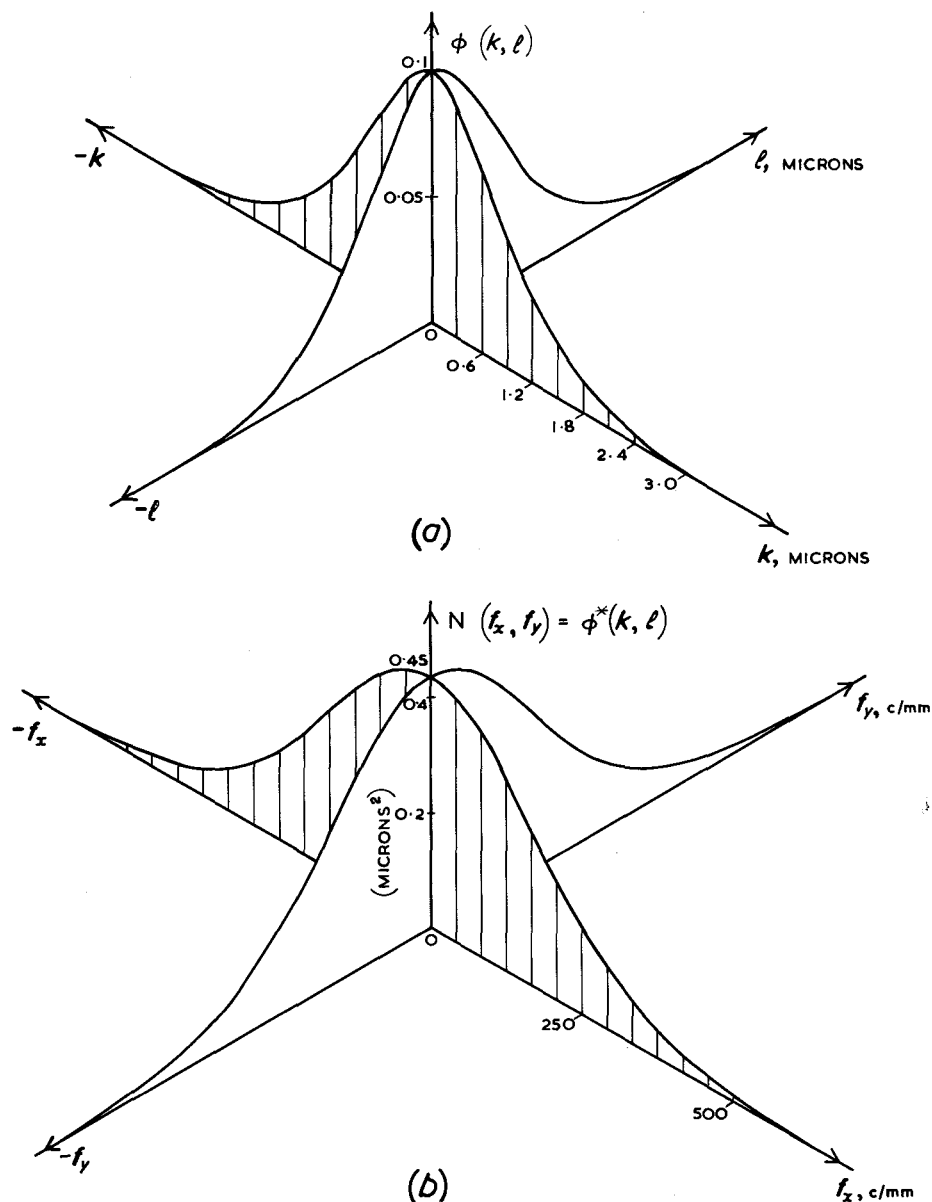


Fig. 4 - (a) Right sections of a typical two-dimensional grain autocorrelation function. Variance = $\phi(0,0) = 0.1$

(b) Right sections of the Wiener spectrum $N(f_x, f_y)$ of the same sample.

$N(f_x, f_y)$ is the Fourier transform of $\phi(k, l)$ and

$$\phi(0,0) = \iint_{(\infty)} N(f_x, f_y) df_x df_y$$

As a basis for comparison with measured autocorrelograms of actual emulsions it is worth analysing a hypothetical model of the grain structure. Suppose that we have a two-dimensional array of circular grains, of equal diameter d , which are randomly distributed in position in the sense that there are no "preferred" spacings and that the presence or absence of a grain is independent of the position of neighbouring grains. Strictly, the latter postulate of randomness cannot be achieved for

grains of finite size unless the overlapping of grains is allowed. However, providing the relative amount of overlap is small (low mean density of the array) one would expect that the only point-to-point correlation that exists in the structure is substantially restricted to the finite area of the individual grains. Hence the shape of the autocorrelation function of the whole array will be nearly identical to that of a single grain. The generating curve of the normalized autocorrelation function of a circular element of diameter d is:

$$\phi(s) = 1/\pi (\theta - \sin\theta) \quad (14)$$

where

$$\theta = 2\cos^{-1}(s/d), \quad (-d \leq s \leq d)$$

Similarly, the Wiener spectrum of the array is nearly identical in form to that of a single element and is described by the normalized generating curve:

$$N(f) = 4 \left\{ \frac{J_1(\pi f d)}{\pi f d} \right\}^2 \quad (15)$$

$$= \phi^*(s)$$

These functions are shown normalized in Figs. 5(a) and 5(b) respectively. The first zero in the Wiener spectrum, see Fig. 5(b), occurs at the spatial frequency $f_c = 1.22/d$, hence as the grain diameter is reduced the Wiener spectrum becomes more uniform over a prescribed low spatial frequency domain.

We can use the autocorrelation function of the circular-grain model to set up a measure of the average grain size (projected area) of actual emulsions. It may be seen from Fig. 5(a), for instance, that the correlation coefficient for a displacement equal to the radius of a grain is approximately 0.39; so that one empirical measure, which may be termed the "equivalent grain diameter", (E.G.D.), is the width of the normalized autocorrelation function of the emulsion at the 0.39 level. The results quoted later in this report are based on this definition. Another measure, which is possibly more realistic, may be obtained in terms of the autocorrelation function of a refined model in which the circular grains are not of equal diameter but have a simple prescribed size frequency distribution, say a normal distribution, of mean diameter d . More realistic still would be the extension to a three-dimensional array of finite depth.

5. APERTURE WEIGHTING AND GRANULARITY TRANSFER

The picture information recorded on photographic film is modified further by the characteristics of one or more subsequent transfer processes and, finally of course, by the characteristics of the human eye. For example, the most common technique employed in photography is the production of a primary negative from which a positive print or transparency is prepared for final viewing. Here the granularity of the positive emulsion itself is combined with that of the negative from which the positive record is obtained, but the negative granularity transferred is a function of the transfer characteristic of the printing process. In television, there is required the further stage of transmitting and displaying the recorded picture. A transfer process containing both linear and non-linear stages is conveniently specified, for

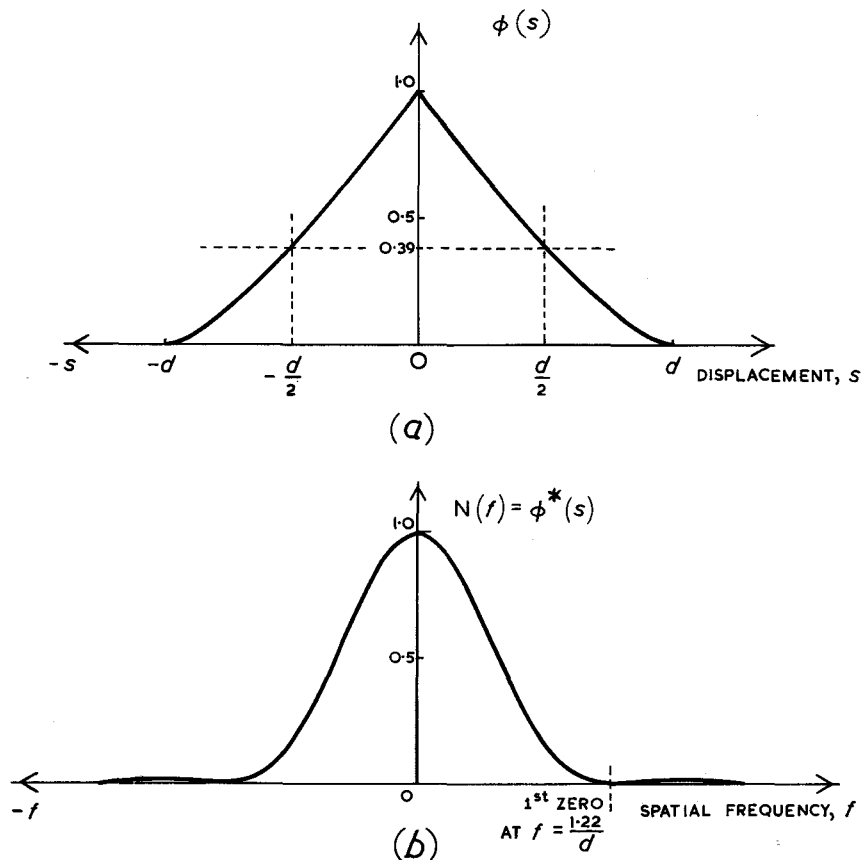


Fig. 5- (a) Normalized generating curve of the autocorrelation function of a (low-density) surface array of circular opaque grains of diameter, d , randomly positioned

(b) Corresponding normalized generating curve of the Wiener spectrum of the model

the purpose of objective analysis, by its complex frequency/response function and the relationship (transfer curve) between the magnitudes of the input and the output. A useful way of assessing objectively the quality of a reproduced television picture with regard to random fluctuation noise is to estimate or determine the r.m.s. magnitude of the luminance fluctuations (at several grey-scale levels) and their Fourier spectrum. This can be done analytically if the magnitude and spectrum of the noise, as well as the transfer characteristics of each stage of the transmission, are known.

In general, the single-stage transfer process involved in photography is non-linear even though the usual aim is to have a substantial degree of linearity in the overall process.

5.1. Effect of Aperture Function

Let us first consider the effect of the frequency/response function of a given linear process on the transferred granularity. Suppose the Wiener spectrum $N(f_x, f_y)$ of a uniformly exposed and developed negative emulsion with mean transmission, $\bar{t}$, has been measured. Let the emulsion be scanned or operated upon with a finite

aperture described by the function $A(x, y)$ whose normalized Fourier spectrum is $V(f_x, f_y)$. The modified or "aperture-weighted" Wiener spectrum $N'(f_x, f_y)$ is now given by:⁵

$$N'(f_x, f_y) = N(f_x, f_y) |V(f_x, f_y)|^2 \quad (16)$$

As shown previously, the variance, σ^2 , of the system is given by the volume under the Wiener spectrum. Hence one may write:

$$\sigma^2 = \iint_{(\infty)} N(f_x, f_y) df_x df_y \quad (17)$$

and for the modified variance σ'^2 :

$$\sigma'^2 = \iint_{(\infty)} N'(f_x, f_y) df_x df_y \quad (18)$$

Or, correspondingly, if the aperture function and Wiener spectrum have rotational symmetry:

$$\sigma^2 = 2\pi \int_0^\infty N(f) f df \quad (19)$$

and,

$$\begin{aligned} \sigma'^2 &= 2\pi \int_0^\infty N'(f) f df \\ &= 2\pi \int_0^\infty N(f) |V(f)|^2 f df \end{aligned} \quad (20)$$

From equations (16), (17) and (18) it is clear that $\sigma'^2 \leq \sigma^2$, the equality sign being applicable when the scanning or operating aperture becomes vanishingly small, i.e., when $A(x, y)$ approaches a Dirac δ function so that $V(f_x, f_y) \rightarrow 1$. Also, when $N(f_x, f_y)$ is substantially uniform over the spatial frequency domain in which $V(f_x, f_y)$ is significantly different from zero we have the condition that σ'^2 varies with the scanning or operating aperture in a manner which is independent of the emulsion. Fig. 6 illustrates how the typical Wiener spectrum of an emulsion is weighted when scanned by a uniform circular aperture (diameter $\simeq 6$ microns): the shaded sections represent the Wiener spectrum of the transferred granularity.

If the above results are combined with the fundamental restrictions for σ^2 given in Section 2 and using equation (2) one obtains, for opaque grains with isotropic statistical distribution:

$$\left. \begin{aligned} \sigma'^2 &= \frac{\sigma^2}{M(\bar{\tau})} \\ &= \frac{\bar{\tau}(1 - \bar{\tau})}{M(\bar{\tau})} \end{aligned} \right\} \quad (21)$$

where,

$$M(\bar{\tau}) = \frac{\int_0^\infty N(f) f \cdot df}{\int_0^\infty N(f) |V(f)|^2 f \cdot df}$$

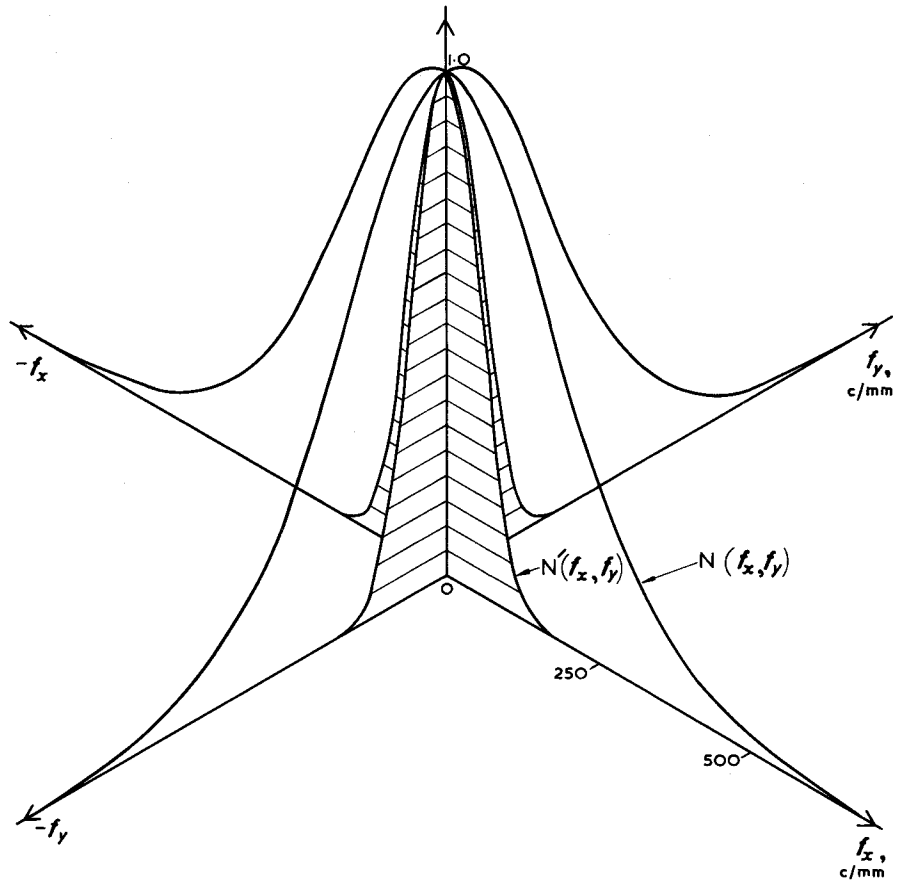


Fig. 6 - Wiener spectrum $N(f_x, f_y)$ of a typical emulsion and the "weighted" Wiener spectrum $N'(f_x, f_y)$ of the transferred granularity when the emulsion is scanned with a uniform circular aperture (Diameter ≈ 6 microns)

$M(\bar{\tau})$ is thus the ratio of the volume under the grain Wiener spectrum to that under the aperture-weighted Wiener spectrum. Note that M is written in equation (21) as a function of the mean transmission, $\bar{\tau}$, on the assumption that the grain Wiener spectrum of actual emulsions may also be a function of $\bar{\tau}$.

Defining the ratio of the modified mean-signal to the r.m.s. noise as $\bar{\tau}/\sigma'$ one obtains, finally,

$$\frac{\bar{\tau}}{\sigma'} = \{M(\bar{\tau})\}^{\frac{1}{2}} \cdot \left\{ \frac{\bar{\tau}}{1 - \bar{\tau}} \right\}^{\frac{1}{2}} \quad (22)$$

A similar expression may be derived in terms of the original and aperture-weighted autocorrelation functions.

5.2. Positive Printing

Suppose that a positive print is made from a uniformly exposed and developed negative emulsion. Let $\bar{\tau}_n$ and $\bar{\tau}_p$ represent the mean transmission of the negative

and positive respectively. $V(f)$ in the third equation (21) now represents the frequency/response function of the negative to positive-exposure transfer process. If it is a contact print $V(f)$ is simply the frequency/response function of the positive emulsion, or if it is an enlargement the frequency/response function of the lens and positive emulsion combination. The transferred or "printed on" variance of the exposure $\sigma_n'^2$, is given by:

$$\sigma_n'^2 = \frac{\bar{t}_n (1 - \bar{t}_n)}{M(\bar{t}_n)} \quad (23)$$

This modified variance exists in the latent image in the positive emulsion and represents the "noise" transferred from the negative. It is now necessary to consider the non-linear transfer characteristic of the printing process, and here complication arises. Suppose, for simplicity, that the macro transfer characteristic can be applied and is represented by the equation:

$$\Delta D_p \approx -\gamma_p \Delta D_n \quad (24)$$

where ΔD_p and ΔD_n are small increments of the optical densities of the positive and negative respectively, and γ_p is the slope (i.e., point gamma) of the conventional H and D curve at the density D_p . From the definition of optical density, (namely, $D = -\log_{10} t$) one obtains for a small increment Δt in transmission:

$$\frac{\Delta D}{\Delta t} \approx \frac{-0.43}{t} \quad (25)$$

Hence, using equations (24) and (25):

$$\frac{-t_p}{\Delta t_p} \approx \frac{1}{\gamma_p} \cdot \frac{t_n}{\Delta t_n} \quad (\Delta t_n \ll t_n) \quad (26)$$

Regarding t_n as the mean transmission coefficient of the negative and the increment Δt_n as the standard deviation of the "printed on" transmission function then, providing Δt_n is small, equation (26) indicates that the variance induced in the developed positive is approximately the transferred variance, represented by $(\Delta t_n)^2$, multiplied by the factor $(\gamma_p \bar{t}_p / \bar{t}_n)^2$. Formally, re-writing equation (26) in the above terms:

$$\sigma_i^2 \approx \left(\frac{\gamma_p \bar{t}_p}{\bar{t}_n} \right)^2 \cdot \sigma_n'^2, \quad (\sigma_n' \ll \bar{t}_n) \quad (27)$$

where σ_i^2 denotes the variance induced in the developed positive. In addition to the induced variance there exists the normal granularity component (and its associated Wiener spectrum) of the developed positive emulsion itself. The problem that now arises is how one should combine the two components in order to assess the granularity of the print. Zweig¹⁷ has suggested that the variances of the two components may be added since they are statistically independent, but the analysis of a simple grain model shows that this procedure cannot be strictly correct for the following reason. If, as previously (Section 2), we make the reasonable assumption that the grains in

the positive emulsion are opaque then we know that, for a given mean transmission, the variance of the transmission function is constant and independent of the spatial distribution of the grains. Since spatial fluctuations of the exposure due to a grainy negative will alter only the spatial distribution of the developed positive grains the variance must remain the same as that of the positive emulsion when uniformly exposed. This does not mean, however, that when the positive print is subsequently scanned with an aperture having a relatively low effective bandwidth the variance of the output signal fluctuations will not be substantially increased, because the print autocorrelation function must be modified by the non-uniformity of the exposure. What must happen, therefore, is that the shape of the print autocorrelation function $\phi(k,l)$ is modified by the non-uniform exposure but its value at $\phi(0,0)$, which is the variance, remains unaltered. Or, in terms of the Wiener spectrum, the latter is modified in shape but the total volume under the spectrum remains unaltered.

Let us proceed with the problem on the assumption that the shape of the print Wiener spectrum is determined by the sum of the Wiener spectrum of the spatial fluctuations of the transmission induced in the positive print by the non-uniform exposure and the Wiener spectrum of the positive emulsion when uniformly exposed. This procedure is illustrated in Fig. 7(a), where the full-line C is the generating curve of the total Wiener spectrum and is the sum of the Wiener spectrum of the induced transmission fluctuations (dashed-line B) and the Wiener spectrum of the positive emulsion when uniformly exposed (dashed-line A). Curve C in Fig. 7(a) thus represents the modified spectrum shape: but in order to comply with the above argument on the constancy of the variance it now requires normalizing to have the same total volume as that of the positive emulsion when uniformly exposed (Curve A). The approximate normalizing factor, β , is given by:

$$\beta \simeq \frac{\sigma_p^2}{\sigma_p^2 + \sigma_i^2} \quad (28)$$

where σ_p^2 is the variance of the positive emulsion when uniformly exposed.

In ordinary positive printing, because of the bandwidth restriction imposed by the "tailing" frequency/response characteristic of the positive emulsion, σ_i^2 will be small compared to σ_p^2 so that β will be close to unity.

The full-line curve in Fig. 7(b) shows the measured Wiener spectrum of a positive-type emulsion (Kodak 5302) exposed through a larger-grained negative emulsion (Kodak Tri-X): the "two-component" structure of the print spectrum is clearly seen. The dashed-line curve in Fig. 7(b) is the expected print spectrum deduced from the measured spectra of the two emulsions when uniformly exposed* and combined according to the approximate relationships outlined earlier in this Section. (The measured sensitometric data used in the calculation are indicated on the Figure: existing data for the frequency/response characteristic of the positive emulsion were used.) It will be noted that the sharp peak at zero frequency predicted by the transfer theory is not fully resolved in the measurement. This is probably attributable to instrumental limitations (some of which are discussed later) rather than to a basic disagreement between experiment and theory.

If the print is now scanned with an aperture having a relatively low effective bandwidth the variance of the output signal fluctuations will be controlled

* See section 8.1.

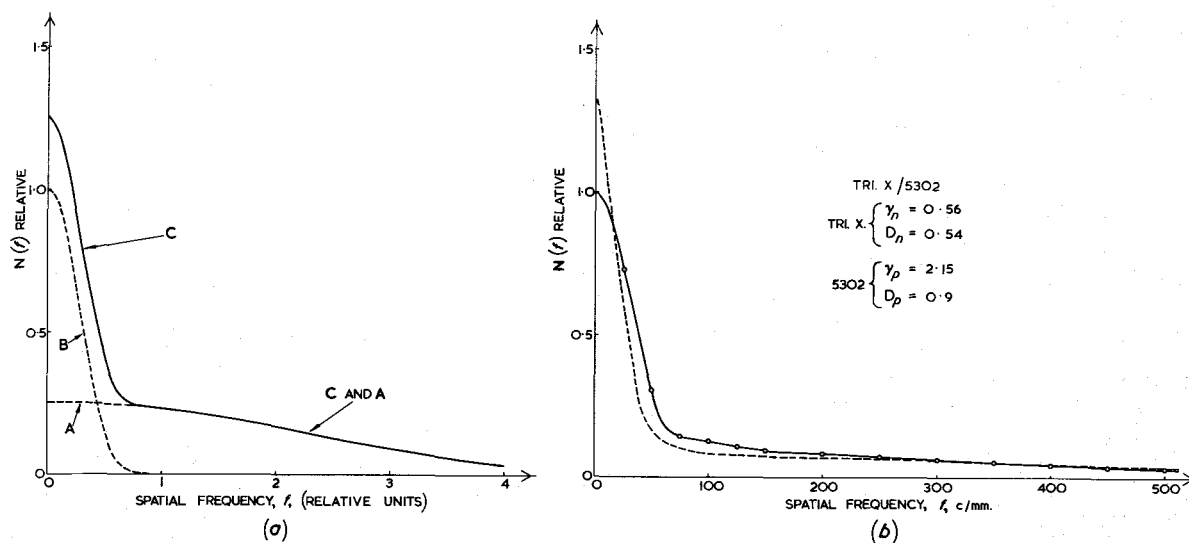


Fig. 7 - Print Wiener spectra

(a) Illustrating the general synthesis

- (Curve A Spectrum of positive emulsion when uniformly exposed
- (Curve B Spectrum of the transferred component of the negative granularity
- (Curve C The sum of A and B spectra to give combined print Wiener spectrum

(b) Actual Wiener spectrum of Kodak Tri-X printed on Kodak 5302. Compare general shape of the spectrum with curve C in Fig. 7(a)

largely by the granularity component transferred from the negative if the mean grain size of the positive is small compared to that of the negative. Reference to the relative levels of the Wiener spectra of the two components at low spatial frequencies shown in Fig. 7(b) will make this point clear.

Positive printing is a practical situation in which the grain Wiener spectrum may not be "flat" even over a relatively narrow range of spatial frequencies. When this condition obtains, the objective assessment of the print granularity by one-parameter measures, such as the Selwyn "granularity constant" may be misleading, because the assessment would be markedly dependent on the size of the scanning aperture.

6. EXPERIMENTAL TECHNIQUE

The aim of the experiment is to determine the autocorrelation functions of uniformly exposed samples of several commonly used photographic emulsions in order to examine the properties of their respective grain structures when processed according to standard commercial practice. From the measured functions their corresponding Wiener spectra are deduced by numerical Fourier transformation. An optical method is used and the first step is to prepare an identical pair of diapositive photographic plates (i.e. transparencies) each being (ideally) a faithful replica of a sample

of the developed grain structure projected normal to the emulsion surface. The identical plates are then placed together in tandem and the average transmission of the pair is recorded as one of the plates is displaced laterally, in a fixed direction relative to the other, starting from the position of zero displacement (i.e., perfect register). If this procedure is carried out for several directions of displacement it will be seen that an approximation to the autocorrelation integral (see equation (7) in Section 4) can be derived. Fig. 8 shows diagrammatically the two plates in tandem with a lateral shift, s , applied to the plate A in the x -co-ordinate direction. The maximum transmission is obtained when the two plates are in perfect register. As one plate is displaced the transmission will decrease and converge to a steady level which corresponds to the product of the mean transmissions of the individual plates.

In order to measure the correlation for displacements much less than the average diameter of the individual grains, and for other reasons, it is convenient to use greatly enlarged diapositive replicas, otherwise the required displacement precision of the autocorrelator becomes difficult if not impracticable to achieve.

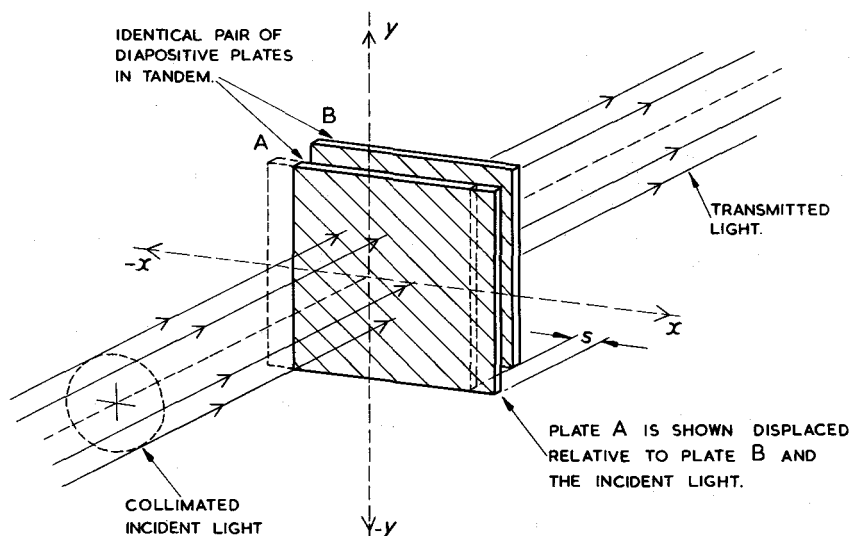


Fig. 8 - Principle of the method of measuring the autocorrelation function

6.1. The Photomicroscope

The apparatus constructed to produce the enlarged diapositive replicas is shown in Fig. 9, and may be described as a simple photomicroscope. A variable-intensity tungsten lamp (L) illuminates the field stop (S) which is imaged by a 16 mm microscope objective (C) on the emulsion sample held flat in the clamp (G). The incident illumination is restricted to a narrow spectral region centred at $500 \text{ m}\mu$ by the metal-dielectric interference filter (F). The main 8 mm objective (O), (with numerical aperture, $NA = 0.65$), together with a simple negative lens (N), projects a magnified image of the illuminated portion of the sample on to one of the photographic plates (P) or (P'). The photographic plate-holder (H) can be moved along guide rails (shown as dashed lines) to vary the overall linear magnification in the range $\times 100$ to $\times 200$. The removable mirror (M), shown dotted, is inserted to

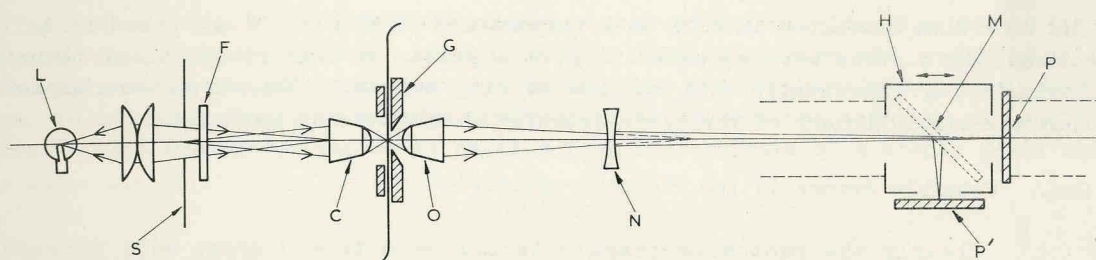


Fig. 9 - Optical arrangement of the photomicroscope

- | | |
|-------------------------------------|------------------------------|
| L - Tungsten lamp | G - Film clamp |
| S - Field stop | N - Negative lens |
| F - Narrow-band interference filter | H - Plate-holder |
| C - Condensing microscope objective | P - P' - Photographic plates |
| O - Imaging microscope objective | M - Front surface mirror |

expose the plate (P'), and removed to expose plate (P). Thus, by making two successive exposures, two negative replicas are produced which are mirror images of each other. A single plate is, in fact, used with the two exposures side by side. After developing the negative plate a positive is produced by contact printing on to a second plate. Fig. 10 shows a typical diapositive pair at this stage. Finally the positive plate is cut in half and the two identical replicas are mounted (emulsion facing emulsion) in the optical correlator.

After several trial experiments it was found that Kodak O-250 orthochromatic plates were suitable for preparing the diapositive pairs. These plates have a fairly uniform point-gamma transfer characteristic over a density range of about 1.5, and a gamma of unity can be obtained with exposure times of about 20 to 30 seconds. The

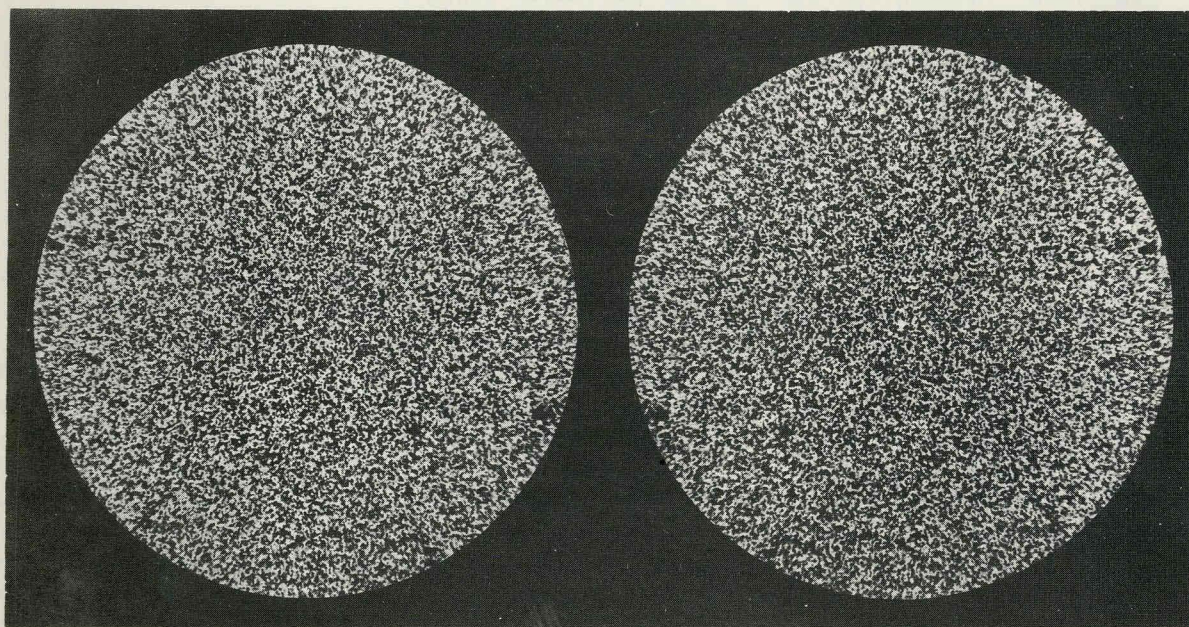


Fig. 10 - A typical mirror-image pair of photomicrographs prepared for autocorrelation measurement

plates were dish-developed in D.76 at a temperature of $69^{\circ}\text{F} \pm 1^{\circ}\text{F}$ and fixed in Amfix. The development times were adjusted to give a gamma, at each stage, close to unity following trial experiments with calibrated step wedges. Exposures were adjusted so that the straight part of the plate transfer characteristic was used.

6.2. Possible Errors in the Photomicrography

Clearly the photomicrographic technique outlined above will introduce experimental error. The enlarged photomicrographs of the sample grain structure will not be absolutely faithful replicas for the following reasons.

6.2.1. Limitations of the Microscope Objective

Although an objective with a numerical aperture sufficiently large to resolve the grain structure can be selected, some reduction in image contrast must occur, especially at the edge boundaries of the grains. Veiling glare arising in the instrument also reduces image contrast. Stated more elegantly, the spatial-frequency/response function of the imaging optics will modify or "weight" the spectrum of the grain structure. Further, although the imaging optics can be adjusted to focus precisely on a particular plane in the emulsion layer, those grains lying some distance from this plane may be out-of-focus. This effect is mitigated to some extent by the fact that the largest proportion of developed grains lies near the exposed surface.

In order to avoid spurious correlation effects, it is desirable to carry out correlation measurements relating to as large an area of the sample as possible, so that the averaging takes place over a large number of grains. In the apparatus used, the flatness of field of the photomicroscope limited the usable size to a circular area approximately 200 microns in diameter. At a mean transmission of 0.5 and a mean grain diameter of 1.5 microns the sample area probably includes some 10^4 grains, although not all of these will be in optimum focus.

6.2.2. Photographic Processing

Here again the spatial-frequency/response function of the plates used to photograph the projected grain structure modifies the true spectrum of the grain sample, but the "weighting" is small because of the large linear magnification employed. A similar argument can be applied to the effect of the plate granularity. A serious distortion of the enlarged replica may occur if the overall negative-positive process is not linear. Strict linearity implies an overall process gamma of exactly unity over the contrast range in the exposure. This is difficult to achieve, and the estimated accuracy obtained in the process gamma was about $\pm 10\%$. It was found by experiment that departures from an overall process gamma of unity of the order of 10% had an insignificant effect on the shape of the autocorrelation function, but the measured variance of the diapositive plates was significantly affected as might be expected. Better control of the gamma can be achieved at the negative stage, and from this point of view it would be preferable to carry out the correlation and variance measurements using a negative pair. There are two objections to this procedure. In the first place the transfer curve (mean transmission versus exposure) at this stage is roughly hyperbolic, which can introduce distortion affecting both the shape of the autocorrelation function and the variance. Secondly,

difficulties arise in deriving the sample signal-to-noise ratio from measurements made on the negative replica. The technique finally adopted was to use positive pairs and correct the measured signal-to-noise ratio by a factor, usually $< 5\%$, derived from the measured change in average grain contrast between the negative and positive pairs. This correction factor is derived from the analysis of a simple grain model and makes use of equation (5) in Section 2.

7. THE OPTICAL CORRELATOR

7.1. Description of the Instrument

The design of the correlator is similar to that used by Kretzmer¹⁸ for analysing photographic images. It is essentially a photo-electric transmission measuring device, with provision for precise registration of a pair of photographic plates and means of producing small increments of lateral displacement of one of the plates.

7.1.1. Optical Design

Fig. 11 shows the basic optical arrangement of the instrument. The photographs to be measured are mounted nearly face to face and placed between a pair of plano-convex lenses, 5 in (127 mm) diameter, 8 in (203 mm) focal length. A small pin-hole aperture situated at the focus of one of the lenses is illuminated by a small (6 V, 3 W) tungsten lamp placed behind a diffusing screen. The collimated light flux which emerges from the photographic plates is focused by the other lens on to the front of a cylindrical light guide. The latter has diffusing material cemented to each end. Behind the light guide is placed the photocathode (2 in (51 mm) diameter) of an eleven-stage photomultiplier tube of the end-window type. A shutter is available to "blank off" the pin-hole light source, and an attenuating (neutral density) filter can be placed over the front of the light guide. The diameter of the collimated pencil is controlled by inserting metal templates, having apertures of the required size, in front of one or both lenses. Each optical "arm" of the instrument is completely shrouded by conical and cylindrical trunking lined on the inside with black flock. The collection angle of the emergent flux is approximately 0.03 steradian: thus the device measures near-specular transmission.

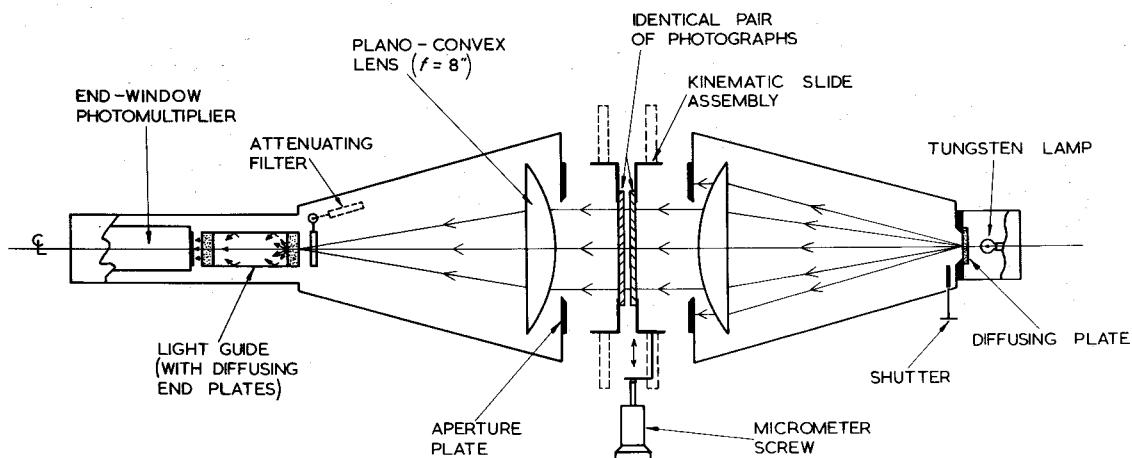


Fig. 11 - Schematic arrangement of the optical correlator (horizontal section)

7.1.2. Mechanical Design

Fig. 12 shows a photograph of the mechanical arrangement. The photographic plates to be measured are mounted emulsion to emulsion, normal to the optical axis of the instrument, and by first attaching them to cylindrical axially sliding plate-holders the emulsion surfaces can be brought close together. Each plate-holder is placed in a sub-assembly which is supported by a kinematic mechanical slide, the pair of slides being independently mounted. Each kinematic slide comprises a Vee section guide with two steel balls for the base and a spring-loaded roller for the top. For initial positioning, the sub-assemblies can be adjusted by sliding them up or down the vertical pillars forming the frame of the kinematic slide, and can also rotate about a common axis parallel to the optical axis. Locking screw controls are provided on each assembly. In order to establish accurate registration of the photographs, fine lead screws are provided for both vertical and rotational adjustment of one sub-assembly relative to the other. After initial registration one of the kinematic slides can be displaced (horizontally) by a known amount by means of a standard micrometer lead screw having a calibrated-drum head. The complete alignment mechanism is mounted on a trolley running along guide rails extending to the front, so that the photographs can be withdrawn (as shown in Fig. 12) from the collimated light beam. This latter facility enables transmission coefficient measurements to be obtained easily and is useful for the loading and pre-adjustment of the assemblies described above.

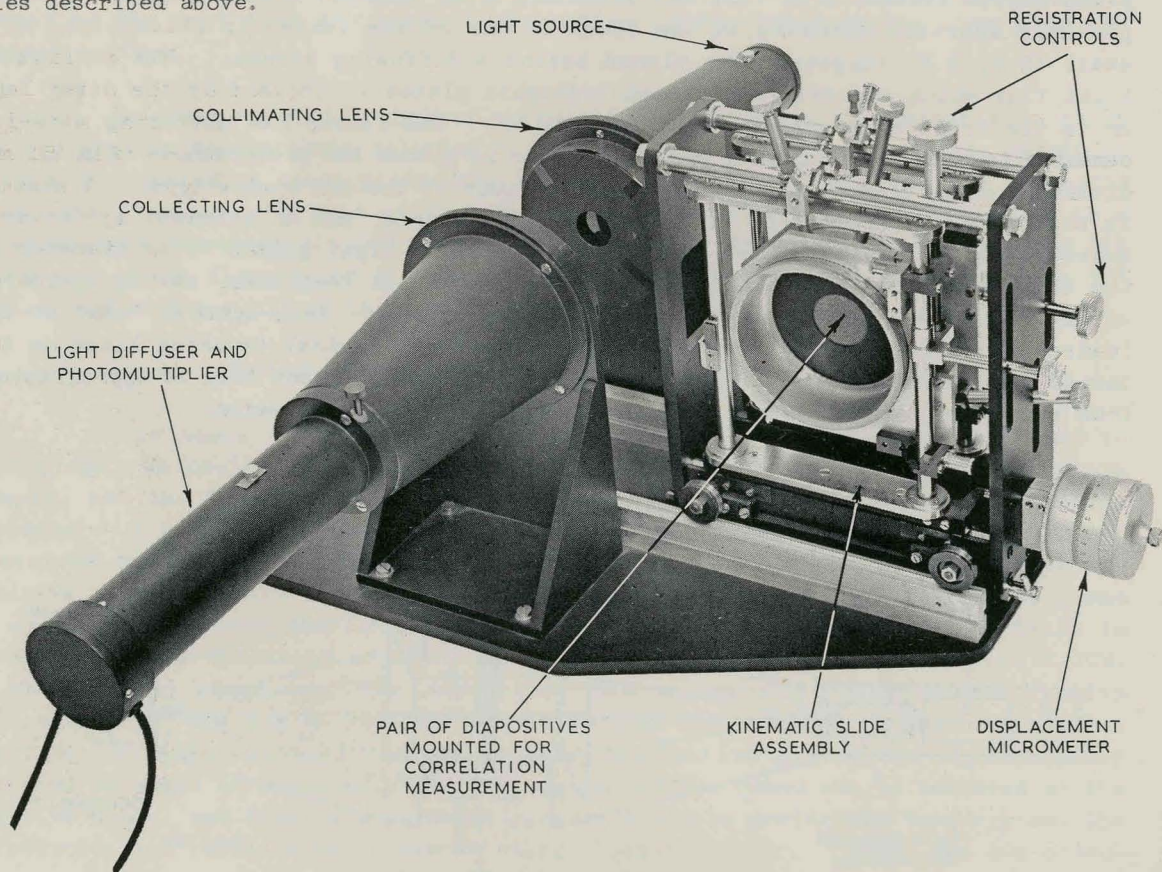


Fig. 12 - Photograph of the optical correlator, showing the mounting of the diapositive pair of plates. The trolley is shown in the position for loading and rough adjustment

7.1.3. Electrical Arrangement

A tungsten light source is run from a 6 V accumulator. A mains rectified 1 kV e.h.t. power supply is used for the photomultiplier, the dynode resistance chain being housed near the base of the tube. Overall dynode voltage stabilization is obtained by a bank of neon stabilizing valves. The signal output of the photomultiplier is coupled to a moving coil "mirror" galvanometer via a control unit. The latter consists of a universal shunt arrangement allowing a $\times 10$ variation in galvanometer sensitivity, a bias circuit, and a potentiometer at the signal input to control the meter current. If high sensitivity to small changes in transmission is required when the standing signal level is large, the bias circuit can be switched in and the standing current "backed" off by means of an opposing current derived from the battery supply to the lamp. Fig. 13 shows a general view of the autocorrelator including the control unit and galvanometer.

7.2. Measurement Procedure

A template having a circular aperture is inserted in front of the collimating lens thus restricting the diameter of the incident light beam: it is usual to choose the beam diameter so that the measured sample area is somewhat smaller than the dimensions of the photographic images recorded. One photograph, mounted on its holder, is placed in one of the mechanical slides and is positioned so that the incident light beam will fall on a suitable region of the photograph. In addition, the plate-holder is rotated in its mount if a particular direction of correlation is required. The second identical photograph is then mounted on the micrometer-controlled slide and roughly registered by visual inspection. All coarse adjustment controls are then locked and the slide trolley pushed into the optical path of the instrument. Final precise registration is carried out by observing the galvanometer



Fig. 13 - General view of the optical correlator with control unit and galvanometer

reading and adjusting carefully the fine adjustment screws in sequence until a maximum output current is obtained. The light source is "blanked off" by the shutter and the galvanometer is set to read zero. (Stray light from the test room is reduced to a small level by appropriate screening.) The shutter is then removed and the input potentiometer is adjusted to give a suitable galvanometer deflection corresponding to the position of registration. The reading of the calibrated micrometer screw which controls the horizontal displacement is recorded. Consecutive increments of displacement, of predetermined values, are introduced and the corresponding galvanometer readings recorded. This is continued until the readings have converged to a sufficiently steady value. After returning the kinematic slide to the recorded position of optimum register the procedure is repeated for displacements in the opposite sense. The recorded data give the relative variation of average transmission of the pair of photographs with lateral displacement. The data can be related to the absolute transmission, if required, by removing the whole slide assembly from the optical path and recording the galvanometer deflection corresponding to the incident flux alone.

Other directions of correlation can be selected and the procedure repeated until the complete autocorrelation function is sufficiently specified.

7.3. Performance Tests

Several experiments were carried out in order to assess the limitations and accuracy of the instrument. The effective spatial uniformity of intensity in the collimated light beam was tested by exploring the cross-section with a small aperture, and noting the variation in galvanometer reading with position. A uniformity within $\pm 3\%$ of the average intensity was obtained over the maximum available beam diameter of 4 in (102 mm). Over the central region of the beam, up to about $1\frac{1}{2}$ in (38 mm) diameter, the uniformity is within approximately $\pm 1\%$. This degree of uniformity was considered to be adequate for most applications.

Pairs of identical templates having simple aperture shapes were made to test the overall performance of the autocorrelator. One pair, designed to test the displacement precision of the kinematic slide, consisted of identical narrow slit apertures, approximately 60 microns wide, made by a vacuum evaporation technique. The result of this test is shown in Fig. 14(a), where the relative transmission of the apertures is plotted against lateral displacement relative to the position of optimum register. The

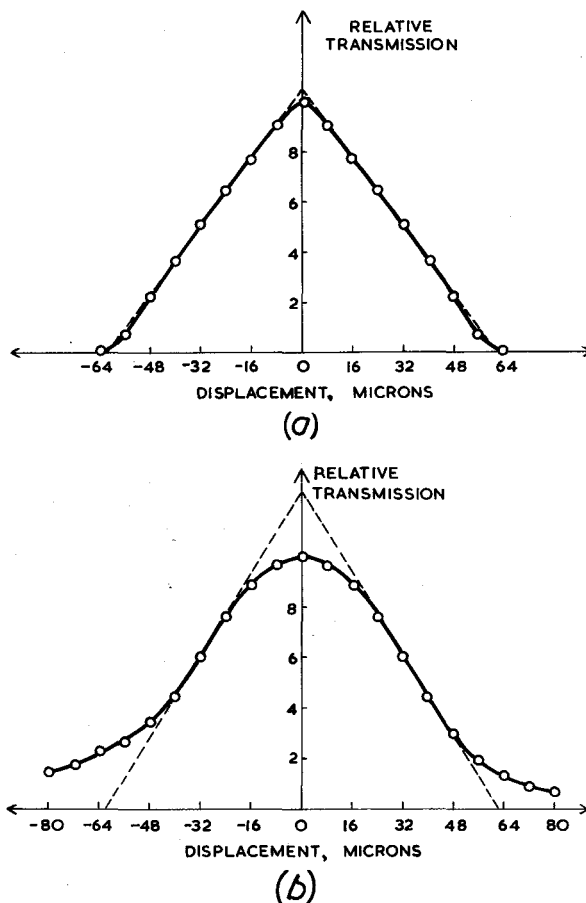


Fig. 14 - Measured autocorrelation function of a slit aperture approximately 60 microns wide

- (a) Axial separation of templates 200 microns
- (b) Axial separation increased to 3 mm

axial separation of the apertures in this test was approximately 0.008 in (200 microns). The theoretical shape of the autocorrelation function for a slit aperture is strictly triangular or roof-topped as indicated by the dashed lines in Fig. 14(a). It will be seen that the measured result exhibits a slight rounding or distortion at both the peak and "skirts" of the function, but the agreement elsewhere is good. The effect of greatly increasing the axial separation of the aperture is shown in Fig. 14(b) where the measurement has been carried out using the same pair of apertures but with an axial separation of 0.125 in (3 mm). For subsequent measurements using photographic plates the axial separation of the emulsion surfaces was adjusted to be less than 0.004 in (100 microns). The incremental displacement accuracy obtainable is estimated to be better than ± 0.5 micron.

7.4. Interpretation of the Measured Function

The recorded function is first made even by taking the mean of the galvanometer readings for corresponding positive and negative displacements. By definition, the autocorrelation function is an even function hence any asymmetry occurring in the recorded data must be due to the pair of photographs not being strictly identical and/or to instrumental errors. A plot of one half of the recorded function at this stage is shown in Fig. 15(a), which refers to a typical result obtained with photomicrographs of emulsion grain. It is seen that the function converges to a substantially constant value which may be estimated by inspection. The estimated constant value, R_s in Fig. 15(a), is taken to be proportional to the square of the mean transmission of the original grain sample. Similarly, the maximum value, R_m , of the function is taken to be proportional to the mean square transmission. That is:

$$(\bar{t})^2 = kR_s$$

and

$$(\bar{t}^2) = kR_m$$

where k is a proportionality constant, from which the variance, σ^2 is given by:

$$\sigma^2 = k(R_m - R_s) \quad (29)$$

and the mean-signal-to-r.m.s. noise ratio, $\bar{t}/\sigma$, is given by:

$$\begin{aligned} \frac{\bar{t}}{\sigma} &= \left\{ \frac{kR_s}{k(R_m - R_s)} \right\}^{\frac{1}{2}} \\ &= \left\{ \frac{R_s}{R_m - R_s} \right\}^{\frac{1}{2}} \end{aligned} \quad (30)$$

The normalized autocorrelation function is derived by subtracting the steady value R_s from the measured function and then normalizing by dividing by $(R_m - R_s)$. Also, in order that the correlation function shall refer to the original sample, the displacement scale is divided by the linear magnification used when preparing the pair of enlarged replicas. Fig. 15(b) shows the normalized autocorrela-

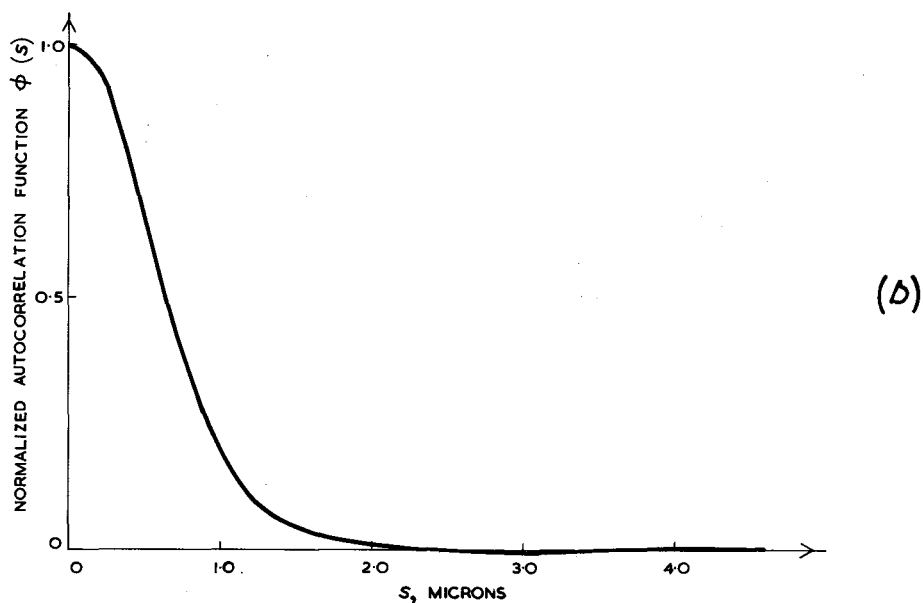
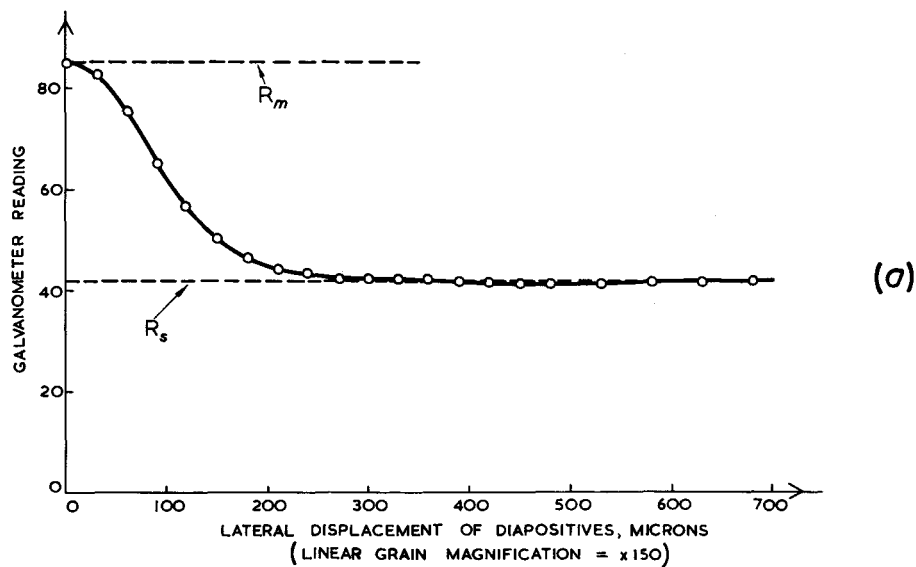


Fig. 15 - (a) Plot of one half of the recorded output of the autocorrelator for a typical magnified grain autocorrelation measurement

(b) The normalized autocorrelation function derived from the recorded data shown in (a)

tion function derived from the measured result shown in Fig. 15(a). If the grain sample is statistically isotropic this curve, Fig. 15(b), represents the generating curve of the two-dimensional autocorrelation function.

The isotropism of the sample can be easily tested by repeating the measurement for other directions of the lateral displacement. In fact, none of the samples tested proved to be significantly anisotropic. Where only small differences in the repeated measurements occur, the generating curve of the autocorrelation function is derived from the mean of the results for two directions at right angles.

8. RESULTS

One or more of six different photographic emulsions (numbered 1 to 6 for convenience) were used for the various experiments outlined below. The emulsion types, gauges, manufacturers' data and typical applications in television are listed in Table 1.

TABLE 1

EMULSION		Film gauge (mm)	MANUFACTURERS' DATA			Television application*
No.	Name		General Description	Speed A.S.A.		
		Tungsten		Daylight		
1	HPS (ILFORD)	35	Panchromatic film of extreme speed.	320	400	Special cine-camera work.
2	Tri-X Type 5223 (KODAK)	35	An extremely high speed panchromatic negative material.	250	320	Stored-field telefilm recording.
3	Plus X Type 4231 (KODAK)	35	A high speed, fine grain panchromatic negative material.	64	80	Standard cine-camera work.
4	Pan F (ILFORD)	35	Panchromatic film of superlatively fine grain.	16	25	35 mm "Quick pull down" and stored-field telerecording.
5	8374 (KODAK)	16	Television recording film. Blue and U.V. sensitive.	-	-	Cable-film and 16 mm telefilm recording.
6	5302 (KODAK)	16	A fine grain release positive film. Blue sensitive.	-	-	Standard positive release printing.

* N.B. These are typical applications of the emulsions at the time of this investigation. Indeed, alternative emulsion types (including more recent developments) which are not investigated here may well be used in many instances.

All the mean transmission values of the developed emulsion samples quoted are relative to that of the base and were measured with the same photomicroscope used to prepare the photomicrographs for the autocorrelation measurements. The following investigations were carried out.

8.1. Comparison of the Autocorrelation Functions and Wiener Spectra of the Six Emulsions

Strips of each emulsion were exposed to tungsten illumination, frame by frame in a camera, so as to obtain, when developed, a graded series of optical densi-

ties on each strip. The emulsions were developed by a commercial film-processing laboratory using standard cine-film processing equipment. The pre-selection of the overall development gamma required was governed by the particular television application for which the emulsion is normally used. Frames which had approximately the same mean transmission ($\bar{t} \approx 0.33$) were selected from each developed strip. A small, blemish-free, area on each frame was selected for producing the photomicrographs required for the autocorrelation measurement. For illustration, photomicrographs of the grain structure of the six emulsions investigated are shown in Fig. 16, where the linear magnification ($\approx \times 400$) is the same for each sample.

The generating curves of the measured autocorrelation functions of the six emulsions are shown in Fig. 17. These curves are normalized to unity at zero displacement and the abscissae scale refers to the plane of the original emulsion. Each curve is derived from the mean of the results obtained for two directions of correlation at right angles, although the differences between the results for the two directions were found to be small.

The mean transmission of the sample, the process control gamma, the measured mean-signal-to-r.m.s. noise ratio and the equivalent grain diameter of each of the six emulsions are given in Table 2.

TABLE 2

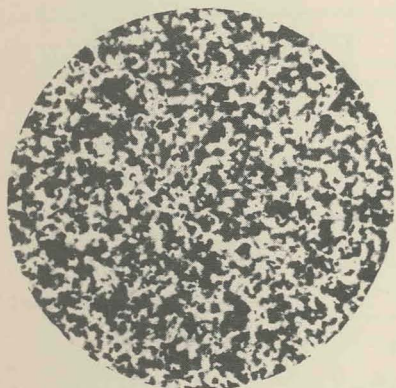
EMULSION		PROCESS CONTROL GAMMA (γ)	MEAN TRANSMISSION OF SAMPLE, $\bar{t}$. (Rel. to base)	MEAN-SIGNAL-TO- r.m.s. NOISE RATIO, $(\bar{t}/\sigma)$, *	EQUIVALENT GRAIN DIAMETER, (microns)
No.	Name				
1	HPS	0.63	0.31	0.96	2.5
2	Tri-X	0.64	0.36	1.02	2.2
3	Plus X	0.64	0.33	1.19	1.45
4	Pan F	1.0	0.37	1.22	1.5
5	8374	1.0	0.33	1.28	1.2
6	5302	2.4	0.30	1.36	1.03

*N.B. The fundamental lower limit for $(\bar{t}/\sigma)$ at a mean transmission of 0.33 is 0.7 (See Fig. 2).

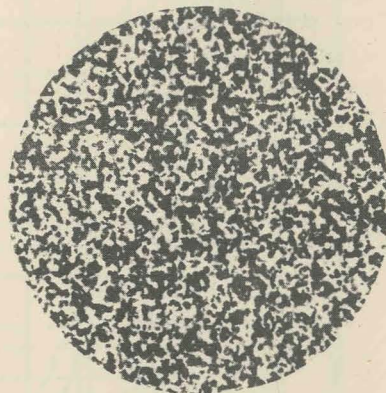
The Wiener spectra of the six emulsions ($\bar{t} \approx 0.33$ for all samples) are shown in Fig. 18, where the ordinate values are in square micron units (μ^2) and the spatial frequency units are cycles/mm (c/mm). The spectra were deduced from the measured autocorrelation functions and variance coefficients of the samples.

8.2. Variation of the Autocorrelation Function, Mean-Signal-to-r.m.s. Noise Ratio and Equivalent Grain Diameter with Mean Transmission

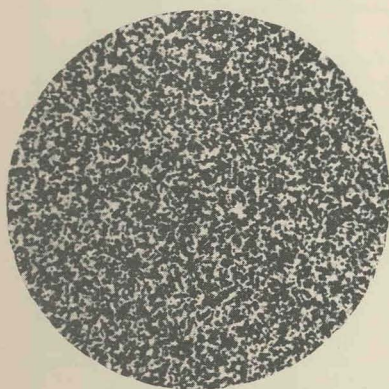
The Pan F emulsion (No. 4, $\gamma = 1.0$) was used for this experiment. The autocorrelation function was measured for each of five samples of the emulsion, the samples having mean transmissions of 0.74, 0.59, 0.37, 0.22 and 0.07 respectively. The normalized autocorrelation functions and corresponding Wiener spectra of these



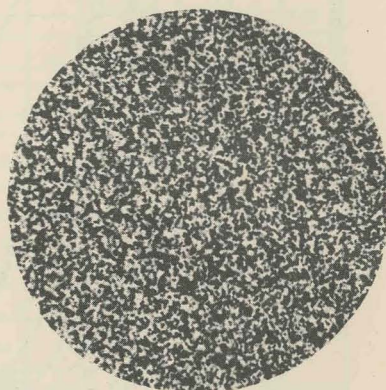
1. HPS. $\gamma = 0.63$



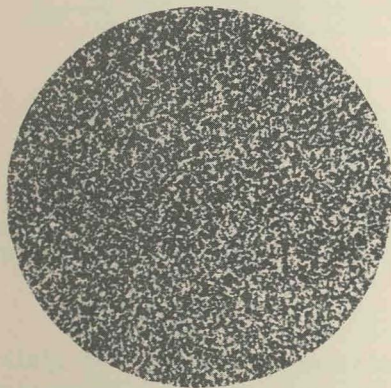
2. TRI.X. $\gamma = 0.64$



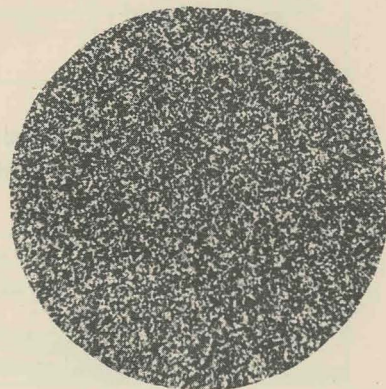
3. PLUS X. $\gamma = 0.64$



4. PAN F. $\gamma = 1.0$



5. "8374" $\gamma = 1.0$



6. "5302" $\gamma = 2.4$

Fig. 16 - Photomicrographs of samples of the six emulsions investigated (at equal mean transmissions, $\bar{\tau} \approx 0.33$). Linear magnification X400

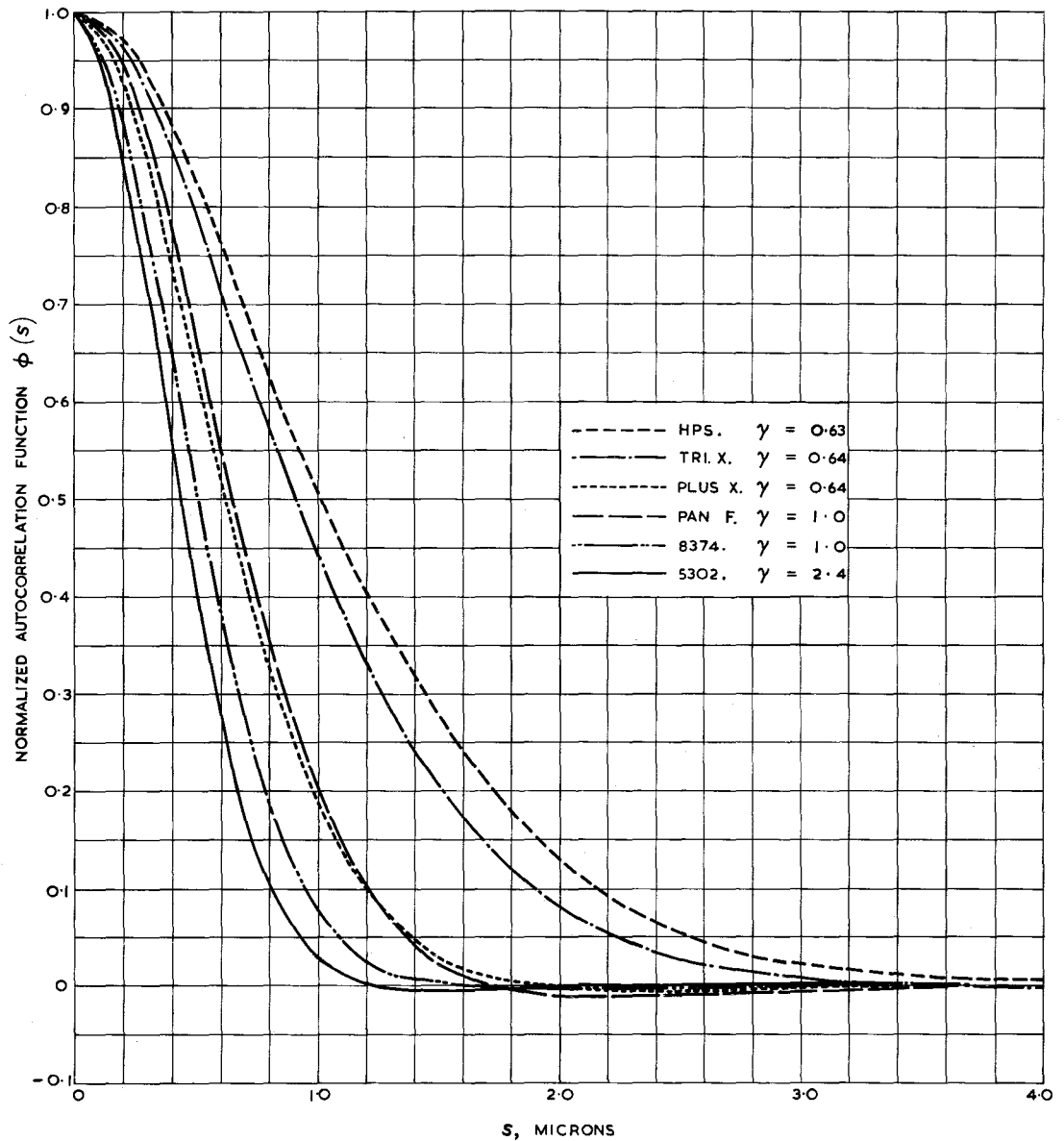


Fig. 17 - Measured autocorrelation functions of samples of six different emulsions at approximately equal mean transmission ($\bar{t} \approx 0.33$)

samples are shown in Figs. 19 and 20 respectively. The Wiener spectra shown in Fig. 20 are now normalized so that the volumes (not areas) under the individual spectra are equal - for easier comparison.

Fig. 21 shows the measured variation of the equivalent grain diameter (full-line) and mean-signal-to-r.m.s. noise ratio (dashed line) with the mean transmission of the sample. It will be remembered that the equivalent grain diameter is the width of the autocorrelation curve delineated by the ordinate value of 0.39, Fig. 5(a) and Section 4.

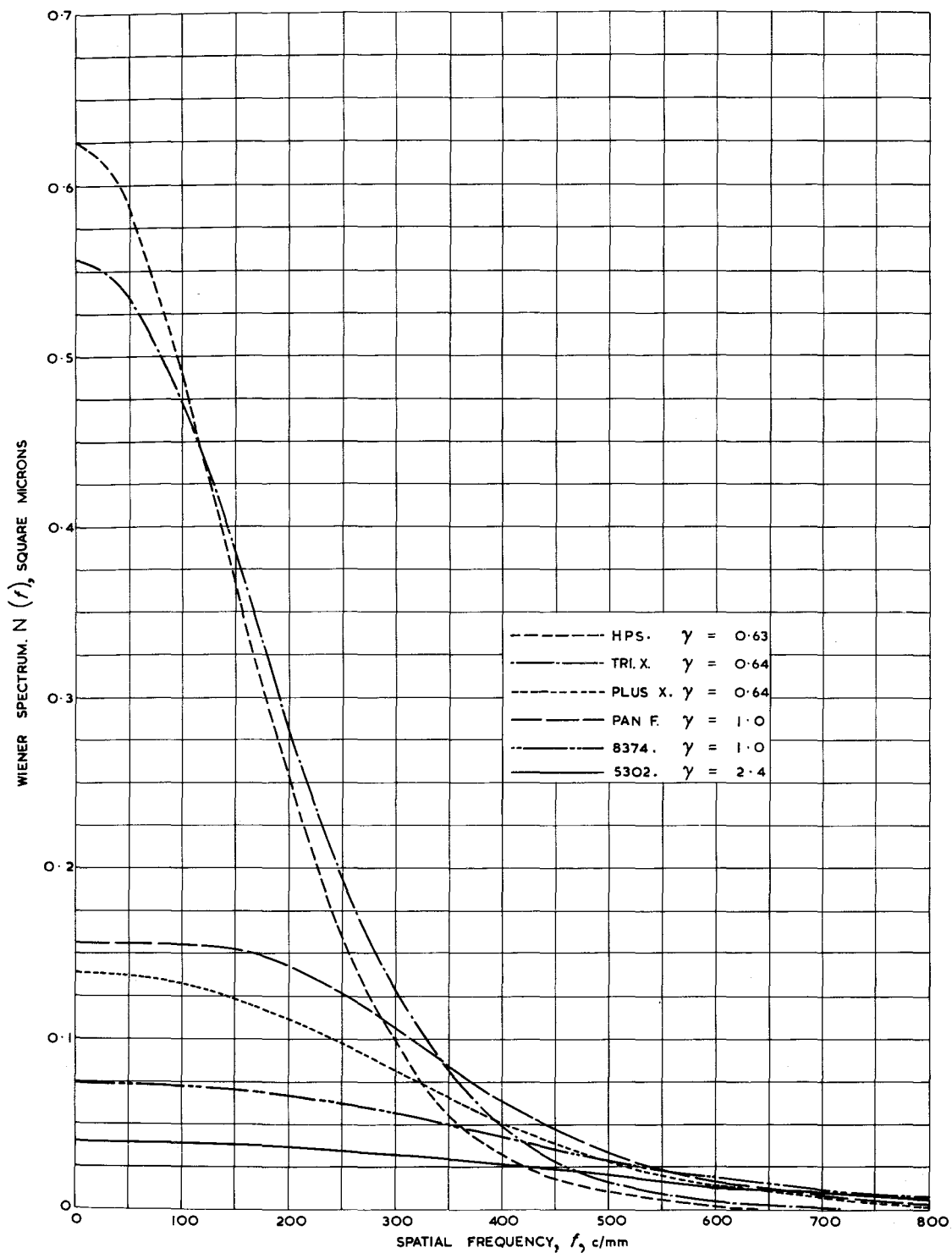


Fig. 18 - Wiener spectra of the six different emulsions: derived by numerical two dimensional Fourier transformation of the autocorrelation functions shown in Fig. 17

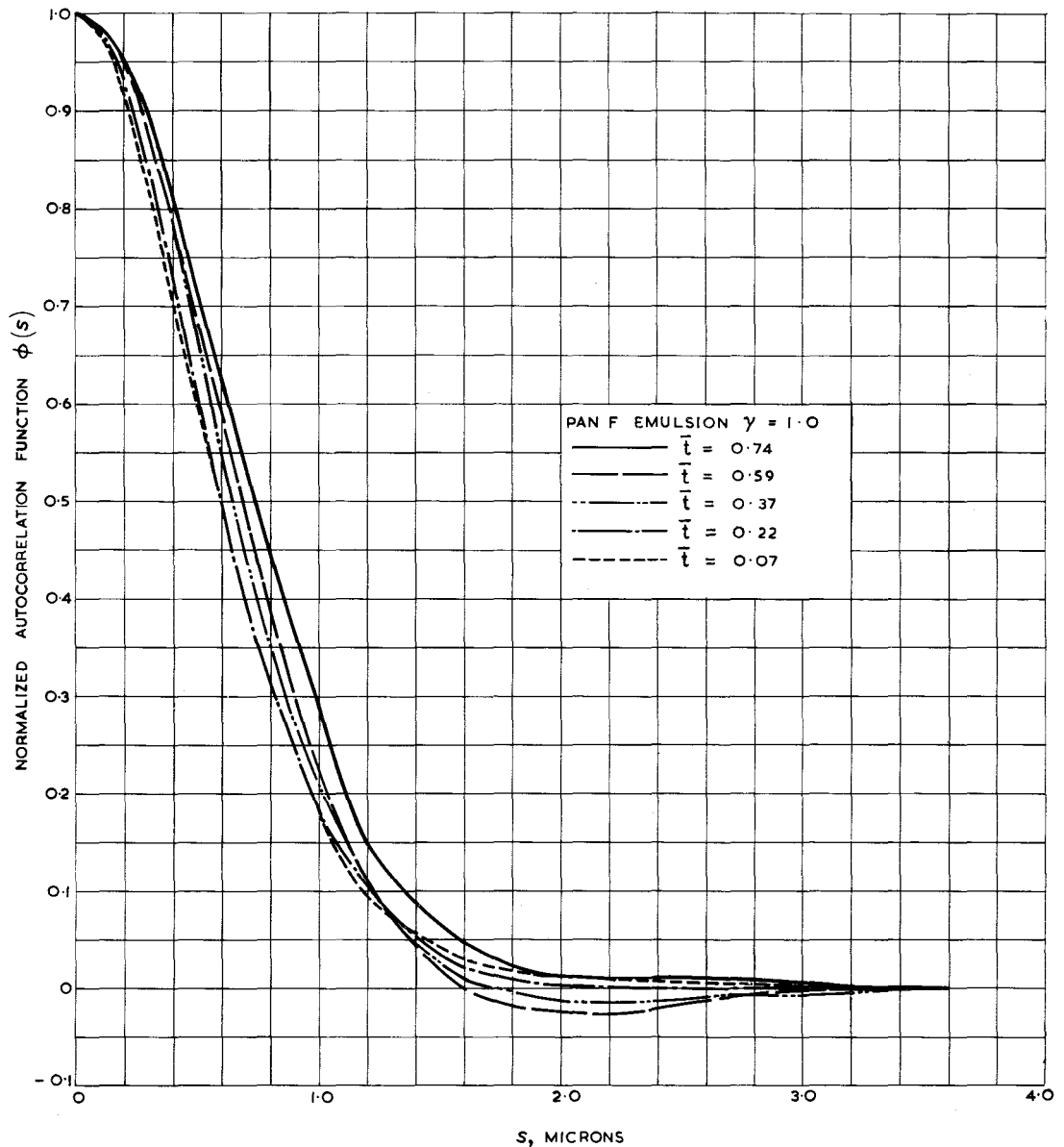


Fig. 19 - Normalized autocorrelation functions of one emulsion (Pan F. $\gamma = 1.0$) for five samples having different mean transmission coefficients, $\bar{t}$

8.3. Effect of Development on Granularity

Test strips of emulsion No. 2 (Tri-X) and emulsion No. 4 (Pan F) were exposed to tungsten light through an optical (grainless) neutral density wedge. The exposed strips, two for each emulsion, were developed in Kodak D.76 solution. By adjusting the development times, to control the slope of the characteristic H and D curve, one strip was developed to a process gamma of approximately 0.6 and the other to a gamma near unity. The actual development characteristic achieved was determined in the normal manner, from density measurements on the exposing wedge and developed emulsion, using an E.E.L. densitometer. For each emulsion, small areas having the same mean optical density were selected and photomicrographs prepared for auto-

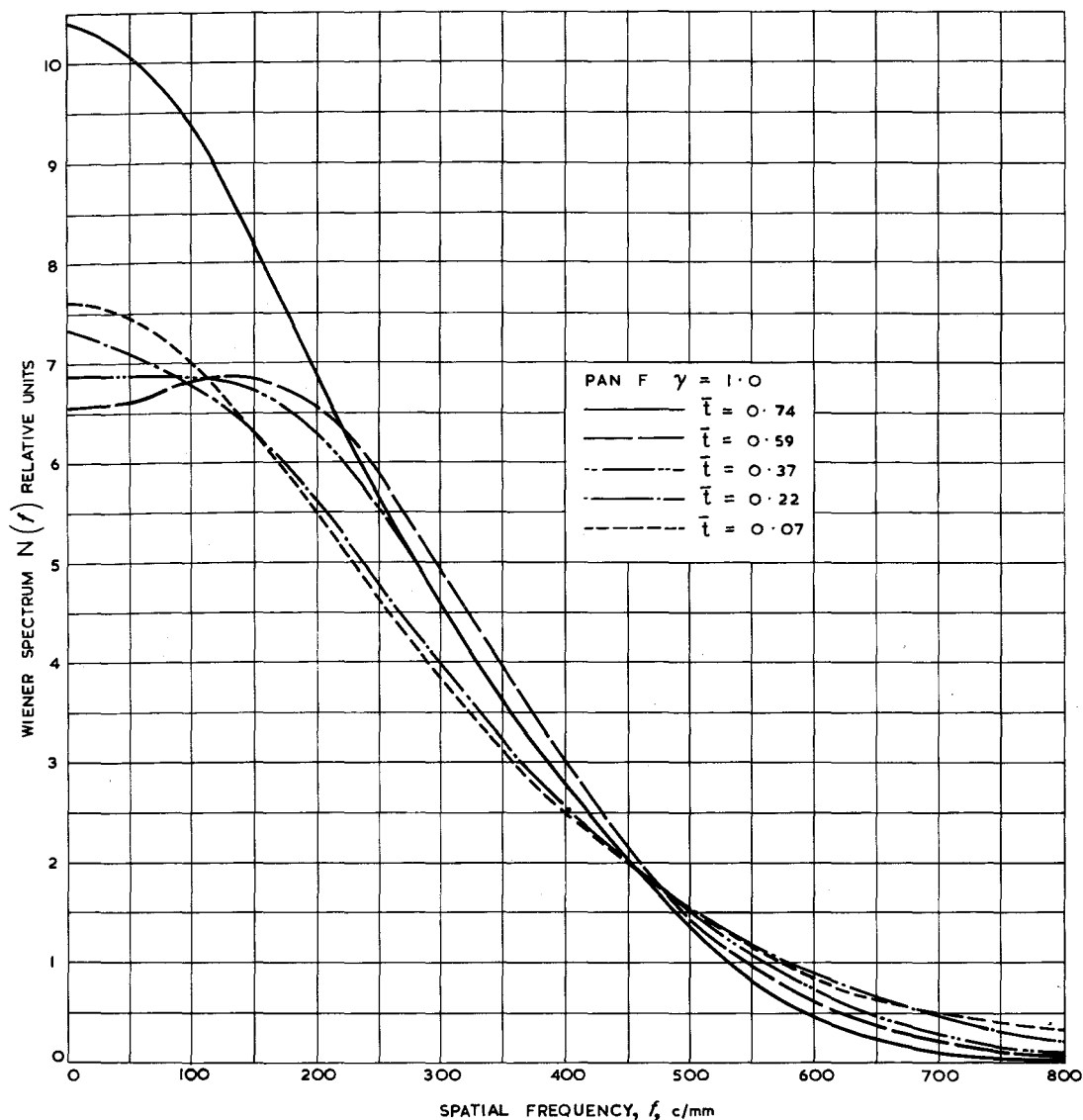


Fig. 20 - Wiener spectra of one emulsion (Pan F. $\gamma = 1.0$)
for five samples having different mean
transmission coefficients, $\bar{t}$. Normalized
for equal volumes under spectra

correlation measurements. The optical densities selected and the slopes or point gammas of their respective development curves at these densities are given, for the two emulsions, in Table 3. Also included in this table are the measured equivalent grain diameter and the ratio of the equivalent grain diameter to the square root of the point gamma for each sample. The effect of the two conditions of development on the developed grain structure is illustrated in Fig. 22, which shows photomicrographs of two samples of the Pan F emulsion both at a mean optical density of 0.7.

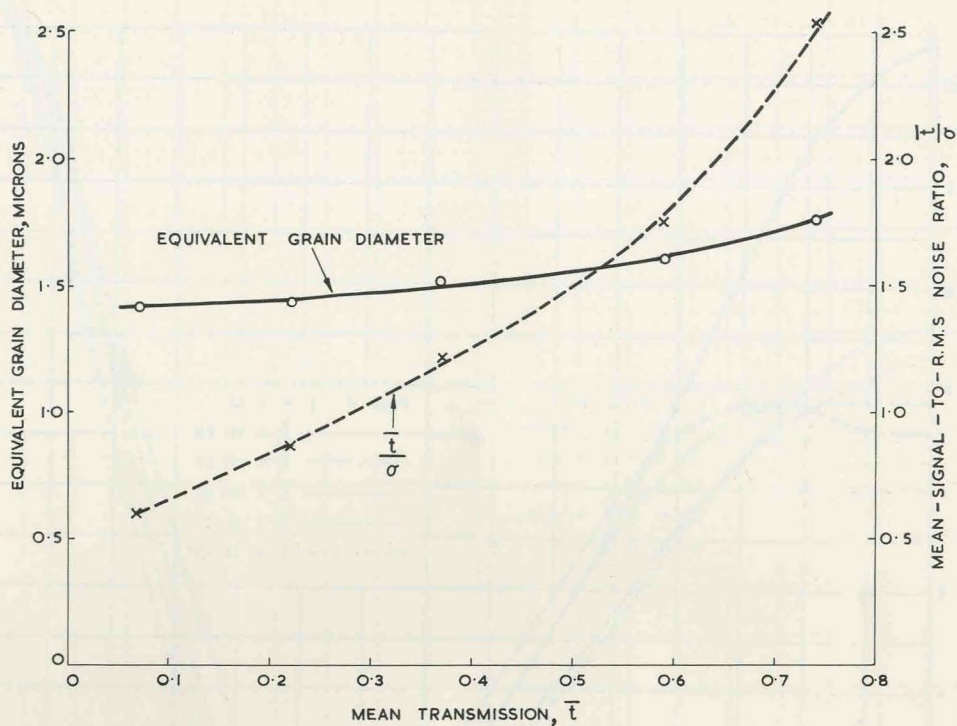
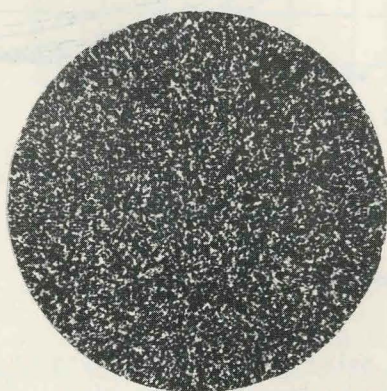
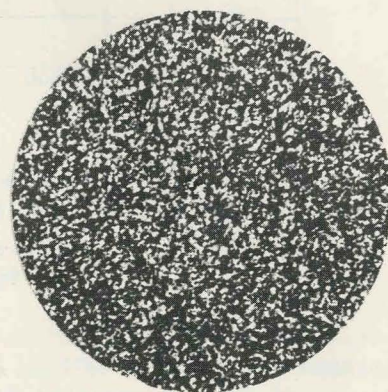


Fig. 21 - Measured variation of equivalent grain diameter and mean-signal-to-r.m.s. noise ratio, $\bar{t}/\sigma$, with mean transmission for one emulsion. (Pan F. $\gamma = 1.0$)

Figs. 23 and 24 show the effect of the variation of the point gamma on the Wiener spectra of the emulsions, Pan F and Tri-X respectively, each at the two densities indicated: in both figures the dashed lines refer to the low-gamma condition of development.



DENSITY = 0.7
GAMMA = 0.63
E.G.D. = 1.1 microns

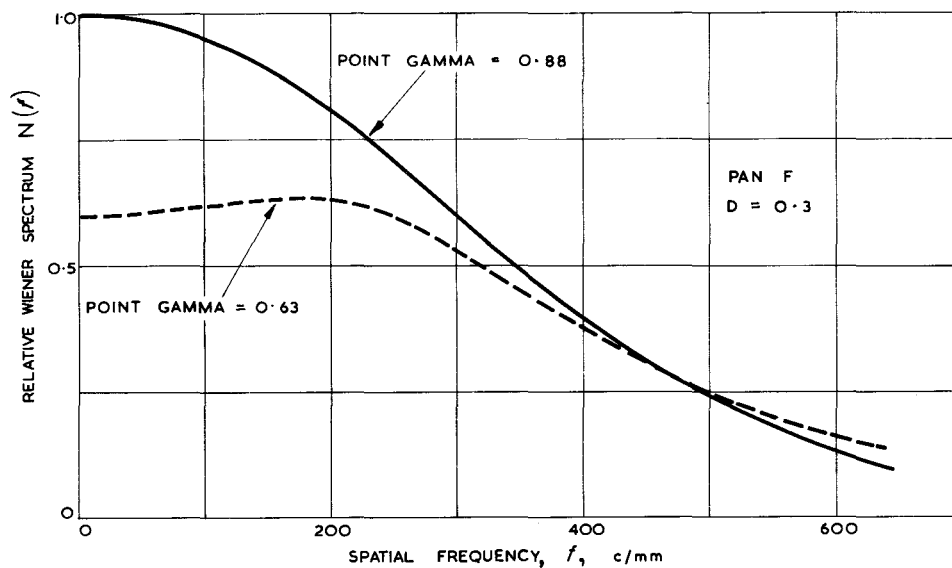


DENSITY = 0.7
GAMMA = 1.1
E.G.D. = 1.4 microns

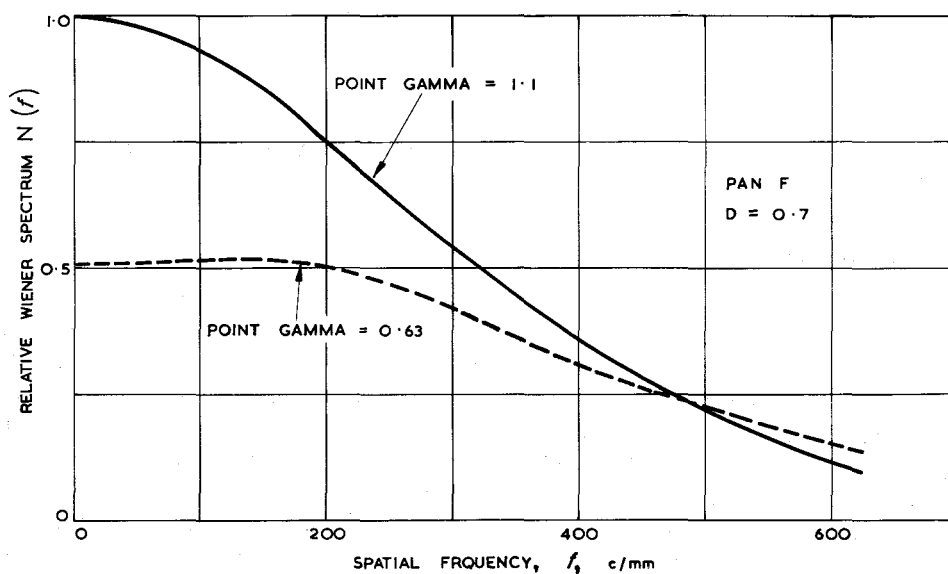
Fig. 22 - Photomicrographs showing the effect of development gamma on the grain size. (Linear magnification x375)

TABLE 3

EMULSION		MEAN DENSITY OF SAMPLE (above base)	DEVELOPMENT GAMMA (γ)	POINT GAMMA	EQUIVALENT GRAIN DIAMETER, (microns)	RATIO OF EQUIVALENT GRAIN DIAMETER TO THE SQUARE ROOT OF THE POINT GAMMA
No.	Name					
2	Tri-X	0.23	0.56	0.45	2.4	3.58
			0.94	0.65	2.72	3.40
		0.54	0.56	0.56	2.12	2.83
			0.94	0.94	2.64	2.73
4	Pan F	0.3	0.63	0.63	1.23	1.55
			1.1	0.88	1.44	1.53
		0.7	0.63	0.63	1.1	1.39
			1.1	1.1	1.41	1.34



(a)



(b)

Fig. 23 - Effect of development "gamma" on the Wiener spectrum of the Pan F emulsion

(a) Mean optical density = 0.3

(b) Mean optical density = 0.7

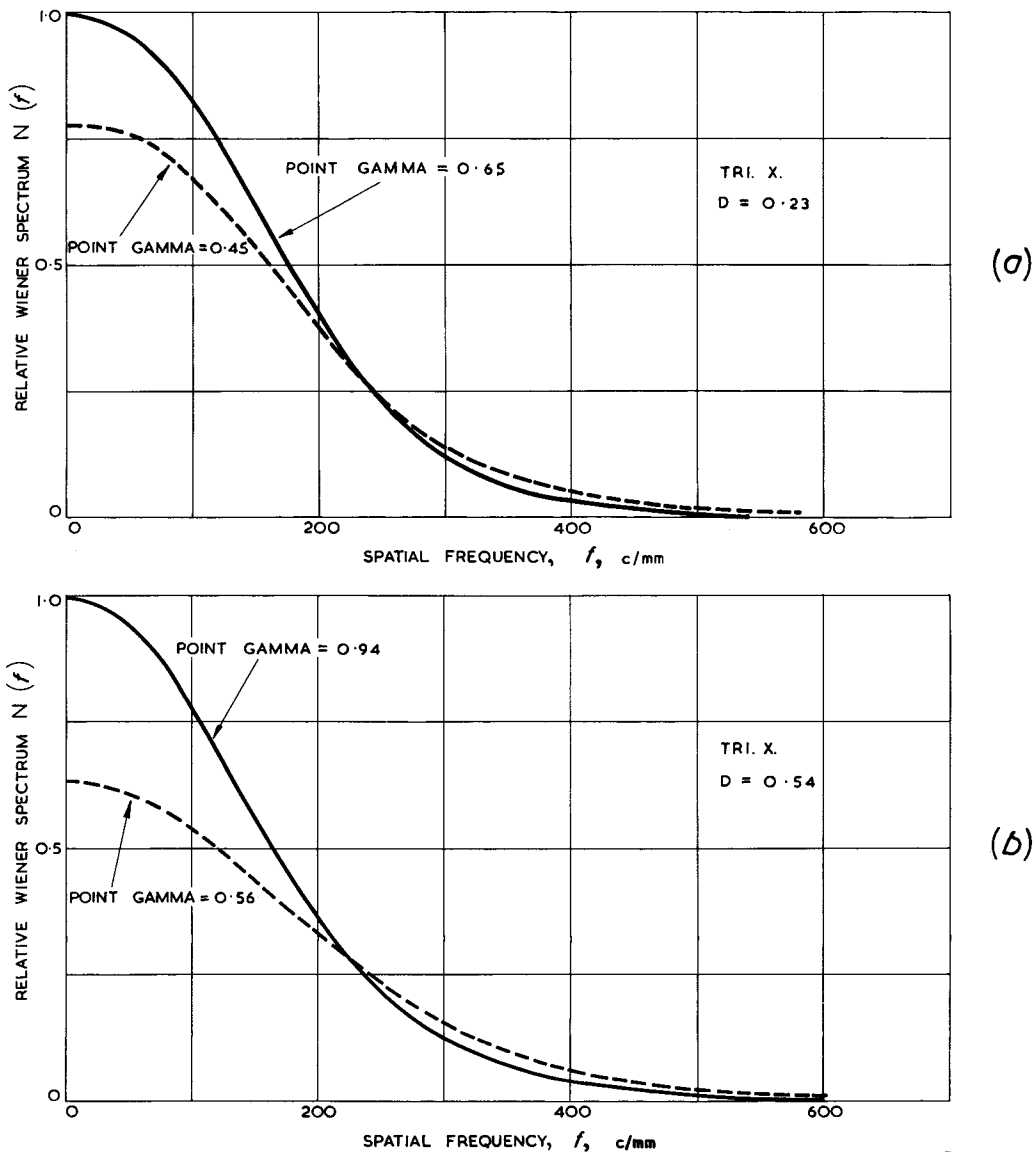


Fig. 24 - Effect of development "gamma" on the Wiener spectrum of the Tri-X emulsion

(a) Mean optical density 0.23

(b) Mean optical density 0.54

8.4. Callier Quotient Measurements

The Callier quotient of the developed test wedges of emulsion No. 2 (Tri-X), prepared for the previous experiment, was measured for a range of optical densities. The method and apparatus used for the measurement was similar to that used by Brandes.¹⁹ Briefly, the apparatus consists of an integrating sphere with an entry window on one radius and a photomultiplier detector, in a side arm, on a radius at right angles to the window position. A small collimated pencil of light (≈ 0.25 in (6 mm) diameter) is projected into the window. The beam is intercepted by the emulsion sample, which is mounted on a saddle free to slide along an optical bench towards the light source. When the sample is close to the entry window of the sphere the total transmitted

flux is collected and, when the sample is some distance from the window only the near-axial or specular transmitted component is received. (The collection angle for the specular component was approximately 0.0016 steradian.) Hence from the transmission measurements for the two positions of the sample the diffuse density and specular density can be deduced. The results are given in Fig. 25(a), where the specular density is plotted against the diffuse density and, as previously, the dashed-line curve represents the low-gamma development conditions. From the above results the Callier quotient is also deduced and Fig. 25(b) shows a plot of the logarithm to the base 10 of the Callier quotient against diffuse density.

9. DISCUSSION OF RESULTS

9.1. Comparison of Results for the Six Emulsions

The first general point is that the measured autocorrelation functions for the six emulsions (Fig. 17) are all of similar form. The functions decrease smoothly, at first resembling a Gaussian function, but with only one exception the functions become slightly negative before converging to the steady level taken to represent zero correlation. This negative "undershoot" is often small (usually $<1\%$) but measureable. It is probable that the actual function converges in an oscillatory manner, but the low amplitudes of the succeeding lobes were not measurable with any certainty. Apart from these

negative excursions, which are discussed later, the functions do not exhibit any "kinks", or "plateaux", or other features which would give rise to marked irregularities in the corresponding Wiener spectrum. As will be seen from Fig. 18 the deduced Wiener spectra for the six emulsions are smooth functions. This is strong evidence for concluding that:

(a) the interseparations of the developed grains in the emulsion layer are highly random in the sense that there are no preferred spacings and that,

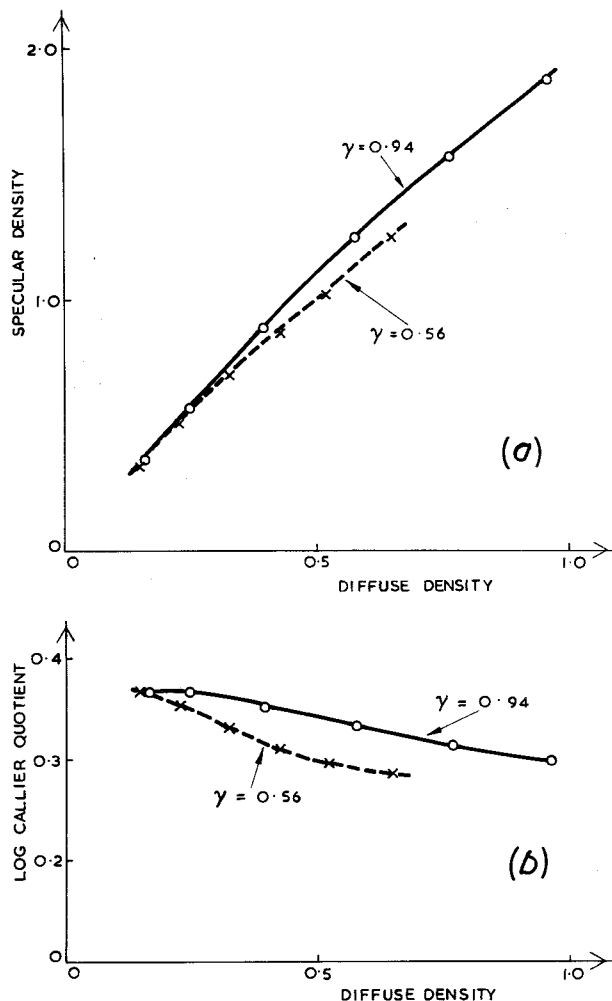


Fig. 25 - Callier quotient measurements on emulsion No. 2 (Tri-X)

- (a) Specular density plotted against diffuse density
- (b) The logarithm of the Callier quotient as a function of diffuse density

(b) the distribution of developed grain sizes* is substantially unimodal, i.e., the size frequency distribution curves which represent the "spread" of grain sizes must contain no pronounced subsidiary peaks.

The point-to-point region of correlation appears to be substantially confined to about plus or minus one equivalent grain diameter, and only small components exist outside this region. The equivalent grain diameters range from 1.03 microns for the fine-grain release positive emulsion (No. 6) to 2.5 microns for the fast cine-negative emulsion (No. 1).

All the results given are uncorrected for the "weighting" effect of the overall measuring process, so that the measured values of the equivalent grain diameters are expected to be greater than the true values - the error being more serious for the fine-grain emulsions. The instrumental "weighting" effect is, also, included in the deduced Wiener spectra shown but, due to the high degree of resolution employed, the "weighting" is harmlessly small at low spatial frequencies.

If we restrict our attention to the low spatial frequency domain, say 0 to 60 cycles/mm, likely to be encountered in ordinary television applications of these films, then the granularity spectra of all the six emulsions can be considered uniform over this range for many practical computations: the spectrum of the HPS emulsion (No. 6), for instance, falling by only about 10% in this frequency range.

From the relative levels shown in Fig. 18 of their Wiener spectra at low spatial frequencies, we may estimate the relative granularity of the various emulsion samples when used with low-bandwidth systems. Describing relative granularity by the relative r.m.s. deviation in transmission for a fixed transmission, the results shown in Table 4 are deduced. Here a bandwidth of 0 to 40 cycles/mm is assumed and the granularity of emulsion No. 6 is taken to be unity. The granularity of the coarsest-grained emulsion measured is seen to be nearly four times that of the finest-grained emulsion. It should be emphasized that a "better or worse" type of comparison is not justifiable at this stage since, in any given application, it is the granularity of the final reproduction which must be judged: the overall process may include the use of two emulsions, or one emulsion processed in a substantially different manner, in order to obtain the required end-result.

TABLE 4

EMULSION		PROCESS CONTROL GAMMA	RELATIVE [†] GRANULARITY
No.	Name		
1	HPS	0.63	3.9
2	Tri-X	0.64	3.5
3	Plus X	0.64	1.8
4	Pan F	1.0	1.9
5	8374	1.0	1.3
6	5802	2.4	1.0

[†] At a mean transmission of approximately 0.33 and bandwidth of 0 to 40 cycles/mm. Emulsion No. 6 taken as unity.

*The term "size" is here intended to mean projected area normal to the layer surface.

9.2. Variation of Granularity with Mean Transmission

There is a small, but definite, decrease in the measured equivalent-grain diameter with decrease in mean transmission. The rate of decrease is largest at high mean transmissions corresponding, of course, to relatively lower exposure levels. One explanation of this trend is the changes which could occur in the distribution of developed grain sizes with exposure level. The larger size grains in an emulsion are, on average, more sensitive than the smaller and have a higher probability of absorbing the requisite number of light quanta to latentize them and thus render them developable. At low levels of exposure, therefore, the size-distribution of latentized grains will be biased further towards the larger grains thus resulting in a larger mean grain size on development. As the exposure level is increased the intrinsic size-distribution of the emulsion will be approached on development.

The measured variation in the ratio $\bar{t}/\sigma$ with $\bar{t}$ agrees well with the variation predicted from the analysis (Section 2, equation (4)) of simple models in which the grains are assumed to be substantially opaque. To compare the measured function with the fundamental lower-limit of the function a further graph was constructed and is shown in Fig. 26, where $\bar{t}/\sigma$ is plotted against the mean optical density ($-\log_{10}\bar{t}$) using logarithmic co-ordinate scales. The full-line curve is the measured variation while the dashed-line curve is the theoretical lower-limit taken from Fig. 2. It will be seen that the slopes of the two curves are approximately the same at the lower end of the range of densities investigated. The higher general level of the measured function is largely due to the fact that the overall measuring process has a limited and non-uniform frequency/response function, so that the measured variance must be smaller than the true variance of the original sample.

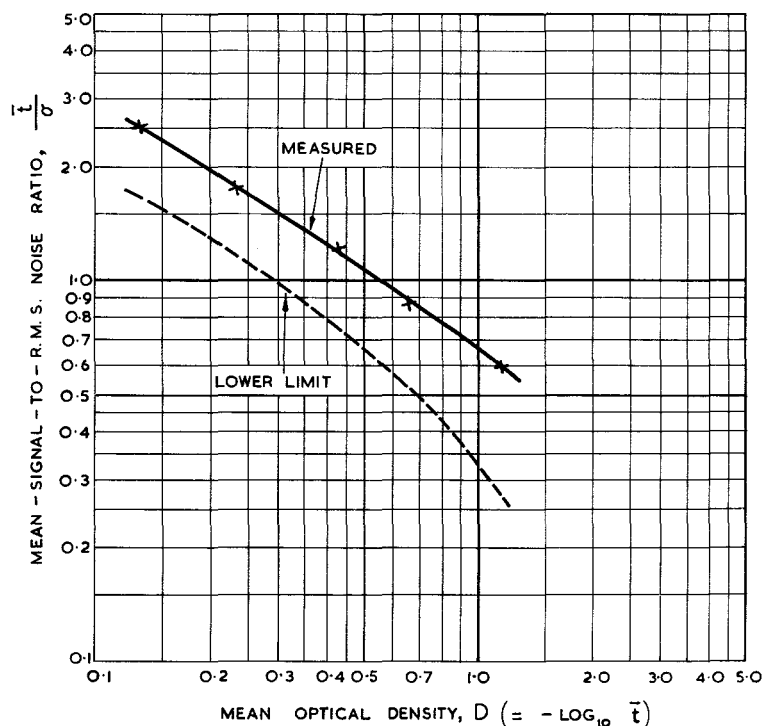


Fig. 26 - Variation of mean-signal-to-r.m.s. noise ratio, $\bar{t}/\sigma$, with mean optical density

The results shown in Figs. 21 and 26 refer to a single emulsion, but as far as can be estimated from measurements on some of the other emulsions the relative variation of $\bar{\epsilon}/\sigma$ with $\bar{\epsilon}$ appears to be substantially independent of the type of emulsion. This, of course, is what one would expect from theoretical considerations for very high-bandwidth applications. For low-bandwidth applications the relative variation of $\bar{\epsilon}/\sigma$ with $\bar{\epsilon}$ will be somewhat modified by any simultaneous change of mean grain size with $\bar{\epsilon}$, thus affecting the level of the Wiener spectrum at low spatial frequencies. However, if these variations in Wiener spectrum are of similar form for all emulsions then the relative variation of $\bar{\epsilon}/\sigma$ with $\bar{\epsilon}$ will, likewise, be substantially independent of the type of emulsion for low-bandwidth applications. The latter condition is substantiated by the work of Higgins and Stultz.¹³

It is valuable to establish an approximate empirical relation between the mean-signal-to-r.m.s. noise ratio and the mean optical density in order to simplify the computation of the granularity transfer between emulsions. From Fig. 26 we find, for extended wide-bandwidth systems, the approximate proportionality:

$$\bar{\epsilon}/\sigma \propto D^{-0.64} \quad (31)$$

where D is the mean optical density. As mentioned above, for low-bandwidth applications it is necessary to take account of the variation in equivalent grain diameter with density because these changes slightly modify the low-frequency portion of the Wiener spectrum of the emulsion. The analysis of a hypothetical circular-grain model (see Section 4) indicates that, for a given mean transmission, the low-frequency level of the Wiener spectrum, and hence the modified variance, $\dot{\sigma}^2$, of a low-bandwidth transfer process, is proportional to the square of the equivalent grain diameter. Using this result and the measured variation in equivalent grain diameter obtained for the Pan F emulsion we deduce the corrected proportionality:

$$\bar{\epsilon}/\dot{\sigma} \propto D^{-0.55} \quad (32)$$

Thus, approximating further for simplicity, one result of this investigation is that the mean-signal-to-r.m.s. noise ratio is inversely proportional to the square root of the mean optical density. The latter result is, in fact, that obtained from a statistical analysis of a three-dimensional opaque-grain model⁷ when the scanning aperture is large compared to the grain size, although it should be pointed out that the statistical analysis assumes that the mean grain size does not vary with density.

From the Callier quotient measurements (Fig. 25(b)) there is some evidence to suggest that the relative variation of equivalent grain diameter with density is also dependent on the development characteristic.

9.3. Negative Correlation

It is possible that the small negative "undershoots" associated with many of the measured autocorrelation functions may be spurious and due to the fact that the sample area used for the correlation measurement was not large enough to include a sufficient number of grains to give a statistical average which is close to that of the parent population. Zweig,²⁰ for instance, reports a similar effect occurring when the autocorrelation function is derived by numerical sampling from recorded microdensitometer-scans if the number of sampling points is insufficient. It may

be noted in Fig. 19, which shows the autocorrelation function of the Pan F emulsion for five different mean transmissions, that the undershoot is more pronounced when the mean transmission is about 50%. However, since the same area of correlation was used for all five of these samples, it could be argued that any spurious correlation effects due to an insufficient number of grains should be more pronounced for the sample with the largest mean transmission where the number of grains is least: but this is not so. Further experiments are required to establish the validity or otherwise of this apparent region of negative correlation. Unfortunately in the present apparatus the maximum sample area is limited by the field curvature of the photomicroscope and the latter must be redesigned to obtain larger fields with a uniformly high resolution.

A small negative undershoot in the correlation function has the effect of "flattening" the Wiener spectrum in the low frequency part of the spectrum: in the most extreme cases encountered a slight minimum at zero frequency was obtained.

9.4. Effect of Development on Granularity

It is clear, from the photomicrographs (Fig. 22) for instance, that the gamma of the development process can have a major influence on the intrinsic granularity of an emulsion. The effect, on the two negative-type emulsions investigated, of increasing the development gamma appears to be solely an increase in the equivalent grain diameter.

For example, with the Pan F emulsion at a mean density of 0.7, development to a gamma of 0.63 produces an equivalent grain diameter of 1.1 microns which places it in the same class as the two fine-grained emulsions (8374 and 5302) measured, while increasing the gamma to 1.1 gives rise to an equivalent grain diameter of 1.4 microns which places it in the medium-grained class. The fact that the variations in equivalent grain diameter and relative Wiener spectra are smaller when the density of the sample is low and therefore near to the toe of the development characteristic suggests that it is the point gamma of the characteristic which is important. If the measured equivalent grain diameter is divided by the square root of the point gamma an approximately constant ratio is obtained for a given mean density, as shown in the last column of Table 3. Hence, for the two emulsions tested, the relative granularity appears to be roughly proportional to the square root of the point gamma of the development characteristic. The effect of different types of developer on granularity has not been investigated, but for equivalent development characteristics the effect is expected to be small.

9.5. Callier Quotient

The ratio of specular density to diffuse density (Callier quotient) is not constant but decreases with increasing density. The Callier quotient is also larger, for a given density, when the emulsion is developed to a higher gamma. As mentioned earlier (Section 3) Eggert and Kuester¹⁵ found that the logarithm of the Callier quotient is proportional to the mean grain size. The results shown in Fig. 25(b) and the variations in equivalent grain diameter with density and development gamma deduced from the autocorrelation measurements appear to be consistent, qualitatively at least, with their findings.

10. CONCLUSIONS

1. The spatial distribution of the developed grains in an emulsion is highly random when uniformly exposed and developed: no significant evidence of grain "clumping" was found in the samples investigated.

2. The granularity spectrum (Wiener spectrum) is determined mainly by the statistical size frequency distribution of the grains. This distribution appears to be substantially unimodal.

3. In the spatial frequency range 0 to 60 cycles/mm, the Wiener spectrum is "flat": that of the coarsest-grained emulsion tested falling by only 10% in this range. The Wiener spectrum of a positive print of a grainy negative (i.e., non-uniform exposure), however, will not in general be "flat" even in this low frequency range (Fig. 7(b)) and the spectrum shape is determined by the spatial frequency/response function of the positive emulsion and the printing optics.

4. For a given emulsion the equivalent grain diameter increases slowly (Fig. 21) with increasing mean transmission (decreasing exposure) suggesting that the size-frequency distribution is modified by the exposure level. If the latter explanation is correct, the rate of variation with mean transmission will depend to some extent on the type of emulsion and may be influenced, also, by the development gamma.

5. An approximate empirical relationship deduced from the results for one negative-type emulsion is that, in the low spatial frequency range, the mean-signal-to-r.m.s. noise ratio is inversely proportional to the square root of the optical density. This relation, however, is expected to be applicable to a wider range of emulsion types.

6. Even for uniform exposures the gamma of the development characteristic can have a pronounced effect on the intrinsic granularity of an emulsion. The randomness of the spatial distribution is not affected but the equivalent grain diameter increases markedly with increasing gamma (Table 3). The results for the two emulsions investigated suggest that for a given developed density the equivalent grain diameter and hence the granularity is approximately proportional to the square root of the point gamma. For this reason it is unrealistic to compare the granularity of, say, two negative-type emulsions when one is developed to a gamma much higher than that of the other, especially when further emulsions are involved in the overall process and/or further non-linear stages precede the image displayed finally.

11. REFERENCES

1. Jones, L.A., and Higgins, G.C., "Photographic Granularity and Graininess",
 J. Opt. Soc. Am., Part I, Vol. 35, p.435, 1945;
 J. Opt. Soc. Am., Part II, Vol. 36, p.203, 1946;
 J. Opt. Soc. Am., Part III, Vol. 37, p.217, 1947;
 J. Opt. Soc. Am., Part IV, Vol. 38, p.398, 1948;
 J. Opt. Soc. Am., Part V, Vol. 41, p.41, 1951;
 J. Opt. Soc. Am., Part VI, Vol. 41, p.64, 1951;
 J. Opt. Soc. Am., Part VII, Vol. 41, p.192, 1951;

2. Goetz, A., and Gould, W.O., "The Objective Quantitative Determination of the Graininess of Photographic Emulsions", J. Soc. Mot. Pict. Eng., Vol. 29, p.510, 1937.
3. Selwyn, E.W.H., "Experiments on the Nature of Graininess", Phot. J., Vol. 79, p.513, 1939.
4. Fellgett, P., "Concerning Photographic Grain, Signal-to-Noise Ratio, and Information", J. Opt. Soc. Am., Vol. 43, p.271, 1953.
5. Clark Jones, R., "New Method of Describing and Measuring the Granularity of Photographic Materials", J. Opt. Soc. Am., Vol. 10, p.799, 1955.
6. Coates, A.C., and Selwyn, E.W.H., Roy. Phot. Soc., Proceedings of Centenary International Conference on the Science and Applications of Photography, London, September 19-25, 1953.
7. Albersheim, W.J., J. Soc. Mot. Pict. Eng., October 1937.
8. Webb, J.H., "Some Theoretical Considerations on Photographic Graininess and Granularity", J. Opt. Soc. Am., Vol. 45, p.379, 1955.
9. Marchant, J.C., and Dillon, P.L.P., "Correlation between Random-Dot Samples and the Photographic Emulsion", J. Opt. Soc. Am., Vol. 51, p.641, 1961.
10. Mees, C.E.K., "Theory of the Photographic Process", (The Macmillan Company, New York, 1948), p.312.
11. See Reference 1, Part II.
12. Pitts, E., and Marriage, A., "Relation between Granularity and Autocorrelation", J. Opt. Soc. Am., Vol. 46, p.1019, 1956.
13. Higgins, G.C., and Stultz, K.F., "Experimental Study of r.m.s. Granularity as a Function of Scanning-Spot Size", J. Opt. Soc. Am., Vol. 49, p. 925, 1959.
14. Frieser, H., and Klein, E., Z. Elektrochem, Vol. 58, p.655, 1954.
15. Eggert, J., and Kuester, A., "Grain Size Determination and Other Applications of the Callier Effect", J. Soc. Mot. Pict. Eng., Vol. 30, p.181, 1938.
16. Loveland, R.P., "Methods of Particle Size Analysis", Symposium on Light Microscopy, Special Technical Publication No. 143, (American Society for Testing Materials, 1952).
17. Zweig, H.J., "Autocorrelation and Granularity, III", J. Opt. Soc. Am., Vol. 49, p.238, 1959.
18. Kretzmer, E.R., "Statistics of Television Signals", The Bell System Technical Journal, Vol. 31, p.751, 1952.
19. Brandes, H., Veroeff Agfa, Vol. 4, p.58, 1935.
20. Zweig, H.J., "Autocorrelation and Granularity, II", J. Opt. Soc. Am., Vol. 46, p.812, 1956.

